(6)

1. $\mathrm{Dom}\, g = \mathbb{R}$

2. $g'(x) = \frac{2}{3}(x^2-1)^{-1/3}\, 2x = \frac{4}{3}\, x\, (x-1)^{-1/3}(x+1)^{-1/3}$

$$\begin{array}{ccccccc} \searrow & & \nearrow & & \searrow & & \nearrow \\ - & -1 & + & 0 & - & 1 & + \end{array}$$

<u>g is decreasing on $(-\infty,-1)$ and on $(0,1)$</u>

<u>g is increasing on $(-1,0)$ and on $(1,\infty)$</u>

(Note, one may NOT take unions here)

By first derivative test, g <u>has local minimums at -1 and 1</u> (note, $g(1)=g(-1)=0$)

and g <u>has local max. at 0</u> (note, $g(0)=1$)

Now, $\lim_{x\to\pm\infty} g(x) = \infty \Rightarrow \sup g = \infty$

$\Rightarrow g$ <u>has no global maxima</u>. Clearly,

$\forall x\ g(x) \geq 0$ and $g(1)=g(-1)=0$. Thus, <u>g has global minima at 1 and -1</u>.

(there are no other global minima, since we have only 2 local minima)

3. $g'' = \frac{4}{3}\left(x(x^2-1)^{-1/3}\right)' = \frac{4}{3}\left((x^2-1)^{-1/3} - \frac{1}{3}x(x^2-1)^{-4/3}\cdot 2x\right) =$

$= \frac{4}{9}(x^2-1)^{-4/3}\left(3(x^2-1)-2x^2\right) = \frac{4}{9}(x^2-1)^{-4/3}(x^2-3)$

$$\begin{array}{ccccccccc} \cup & & \cap & & \cap & & \cap & & \cup \\ + & -\sqrt{3} & - & -1 & - & 1 & - & \sqrt{3} & + \end{array}$$

$g(\pm\sqrt{3}) = 4^{1/3}$

Thus <u>g is convex on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$</u>

<u>g is concave on $(-\sqrt{3}, -1)$, $(-1, 1)$, $(1, \sqrt{3})$</u>

(don't take unions! g' and g'' DNE at ± 1)

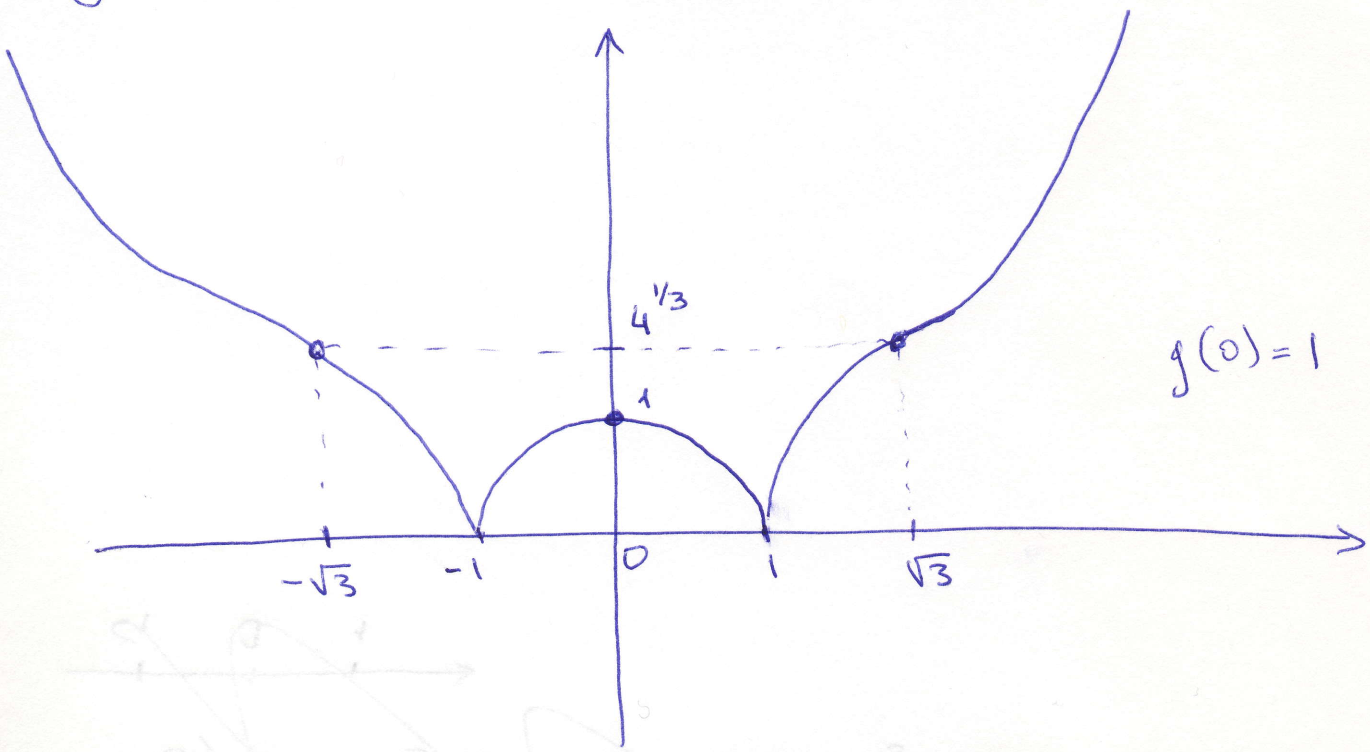
<u>The inflection points are $(\pm\sqrt{3}, 4^{1/3})$</u>

4. $\forall a \quad \lim_{x \to \infty} g(x) = g(a) \neq \pm\infty, \Rightarrow$ <u>no VA</u>

$\lim_{x \to \pm\infty} g(x)/x = \lim_{x \to \pm\infty} \left(\frac{x^2 - 1}{x^{3/2}}\right)^{2/3} = \infty \Rightarrow$ no slant

or horizontal asymptotes $\Rightarrow$

<u>g has no asymptotes</u>



$g(0) = 1$

Note also, g is even $(g(-x) = g(x))$