Problem 1a

1. $\frac{x}{x-5} \geq 0 \iff \begin{cases} x \geq 0 \\ x > 5 \end{cases}$ or $\begin{cases} x \leq 0 \\ x < 5 \end{cases} \iff$

$\iff x > 5$ or $x \leq 0$. Thus

$\underline{\text{Dom } f} = \{x \mid x \neq 5, \frac{x}{x-5} \geq 0\} = (-\infty, 0] \cup (5, \infty)$

2. $f' = \frac{1}{2}\left(\frac{x}{x-5}\right)^{-1/2} \frac{x-5-x}{(x-5)^2} = -\frac{5}{2}\left(x^{-1}(x-5)^{-3}\right)^{1/2} < 0$

Thus f is <u>decreasing</u> on <u>$(-\infty, 0]$</u> and on <u>$(5, \infty)$</u> (Note f is <u>NOT</u> decreasing on $(-\infty, 0] \cup (5, \infty)$)

Since f is diff. on $(-\infty, 0) \cup (5, \infty)$ and $f' \neq 0$, <u>f has no local extrema</u>

$\lim_{x \to 5^+} f(x) = \infty \Rightarrow \sup_{\text{Dom } f} f = \infty \Rightarrow$ <u>f has no global maximum</u>. Clearly, $f \geq 0$ and $f(0) = 0$, so <u>f has global minimum at 0.</u>

3. $f'' = -\frac{5}{2}\frac{1}{2}\left(x^{-1}(x-5)^{-3}\right)^{-1/2}\left(-x^{-2}(x-5)^{-3} - 3x^{-1}(x-5)^{-4}\right) =$

$= \frac{5}{4}\left(x^{-1}(x-5)^{-3}\right)^{-1/2} x^{-2}(x-5)^{-4}\left(x-5+3x\right)$

Thus $f'' \geq 0$ iff $4x - 5 \geq 0$ $\overset{\cap}{-}\ 0\ \ 5/4\ \ \overset{\cup}{5+}$

<u>f is convex on $(5, \infty)$</u>

<u>f is concave on $(-\infty, 0)$</u>

<u>There is no inflection points.</u>

4. If $a<0$ or $a>5$ then $\lim_{x\to a} f(x)=f(a)$.

$\lim_{x\to 0^-} f(x) = 0$, $\lim_{x\to 5^+} f(x) = \infty$. Thus

<u>$x=5$ is a vertical asymptote</u>

$\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to -\infty} f(x) = 1$. Thus

<u>$y=1$ is a horizontal asymptote</u>

<u>No slant asymptotes</u> (since hor. asym. exists)