

Problem 1a

$$1. \quad \frac{x}{x-5} \geq 0 \Leftrightarrow \begin{cases} x \geq 0 \\ x > 5 \end{cases} \quad \text{or} \quad \begin{cases} x \leq 0 \\ x < 5 \end{cases} \Leftrightarrow$$

$\Leftrightarrow x > 5$ or $x \leq 0$. Thus

$$\underline{\text{Dom } f} = \{x \mid x \neq 5, \frac{x}{x-5} \geq 0\} = (-\infty, 0] \cup (5, \infty)$$

$$2. \quad f' = \frac{1}{2} \left(\frac{x}{x-5}\right)^{-1/2} \frac{x-5-x}{(x-5)^2} = -\frac{5}{2} (x^{-1}(x-5)^{-3})^{1/2} < 0$$

Thus f is decreasing on $(-\infty, 0]$

and on $(5, \infty)$ (Note f is NOT

decreasing on $(-\infty, 0] \cup (5, \infty)$)

Since f is diff. on $(-\infty, 0) \cup (5, \infty)$

and $f' \neq 0$, f has no local extrema

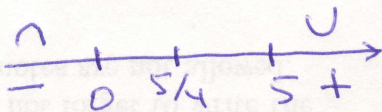
$\lim_{x \rightarrow 5^+} f(x) = \infty \Rightarrow \sup_{\text{Dom } f} f = \infty \Rightarrow$ f has no

global maximum. Clearly, $f \geq 0$ and

$f(0) = 0$, so f has global minimum at 0.

$$3. \quad f'' = -\frac{5}{2} \frac{1}{2} (x^{-1}(x-5)^{-3})^{-1/2} (-x^{-2}(x-5)^{-3} - 3x^{-1}(x-5)^{-4}) =$$

$$= \frac{5}{4} (x^{-1}(x-5)^{-3})^{-1/2} x^{-2}(x-5)^{-4} (x-5+3x)$$

Thus $f'' \geq 0$ iff $4x-5 \geq 0$ 

f is convex on $(5, \infty)$

f is concave on $(-\infty, 0)$

There is no inflection points.

4. If $a < 0$ or $a > 5$ then $\lim_{x \rightarrow a} f(x) = f(a)$.

$\lim_{x \rightarrow 0^-} f(x) = 0$, $\lim_{x \rightarrow 5^+} f(x) = \infty$. Thus

$x = 5$ is a vertical asymptote

$\lim_{x \rightarrow \infty} f(x) = 1$, $\lim_{x \rightarrow -\infty} f(x) = 1$. Thus

$y = 1$ is a horizontal asymptote

No slant asymptotes (since hor. asymp. exists)

