

Remark.

This is drill problems to material which will be covered on Monday, February 22. However, you can start to do them now. You need to know derivatives of exponential and logarithmic functions, which we have already briefly discussed, namely

$$(e^x)' = e^x, \quad (\ln x)' = \frac{1}{x},$$

and that any expression of the type f^g should be reduced to expression

$$e^{g \ln f} \quad \text{indeed} \quad f^g = e^{\ln(f^g)} = e^{g \ln f}.$$

In particular, using this and chain rule,

$$(a^x)' = (e^{x \ln a})' = e^{x \ln a} \ln a = a^x \ln a,$$

$$(x^x)' = (e^{x \ln x})' = e^{x \ln x} (x \ln x)' = x^x (\ln x + x/x) = x^x (1 + \ln x) = x^x \ln(ex),$$

$$(\log_a x)' = \left(\frac{\ln x}{\ln a} \right)' = \frac{1}{x \ln a}.$$

Logarithmic differentiation means that instead of differentiating $F(x)$ we differentiate its logarithm (or, better, logarithm of its absolute value) and use chain rule:

$$(\ln |F(x)|)' = \frac{F'(x)}{F(x)},$$

thus

$$F'(x) = F(x)(\ln |F(x)|)'.$$

For example using this we observe,

$$(x^x)' = x^x (x \ln x)'$$

(we don't pass to absolute value here, since x^x is defined only for $x > 0$.)