Quiz # 4

Problem 1. Provide definitions of $\sinh x$, $\cosh x$.

Definition.

$$\sinh x = \frac{e^x - e^{-x}}{2}, \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

Problem 2. Differentiate

$$(\sin x)^x .$$

Solution. Recall $\exp(x) = e^x$ and $\exp(\ln x) = x$.

$$\frac{d}{dx}(\sin x)^x = \frac{d}{dx}\exp\left(\ln(\sin x)^x\right) = \frac{d}{dx}\exp\left(x\ln(\sin x)\right)$$

$$= \exp\left(x\ln(\sin x)\right)\ \frac{d}{dx}\left(x\ln(\sin x)\right) = (\sin x)^x\ \left(\ln(\sin x) + x\ \frac{\cos x}{\sin x}\right).$$

Answer.

$$\frac{d}{dx}(\sin x)^x = (\sin x)^x\ \left(\ln(\sin x) + x\ \cot x\right).$$

Problem 3. Integrate

$$\textbf{a.}\ \int \frac{\ln^2 x}{x}\ dx, \qquad \textbf{b.}\ \int \arctan x\ dx.$$

Solution.

$$\textbf{a.}\ \int \frac{\ln^2 x}{x}\ dx = \left[u = \ln x \quad du = \frac{dx}{x}\right] = \int u^2\ du = \frac{u^3}{3}\ +\ C = \frac{\ln^3 x}{3}\ +\ C.$$

$$\textbf{b.}\ \int \arctan x\ dx = \begin{bmatrix} u = \arctan x & du = \frac{dx}{1+x^2} \\ dv = dx & v = x \end{bmatrix} = x\arctan x - \int \frac{x}{1+x^2}\ dx$$

$$= \left[t = 1 + x^2 \quad dt = 2x\ dx\right]$$

$$= x\arctan x - \int \frac{dt}{2t} = x\arctan x - \frac{1}{2}\ \ln|t| + C = x\arctan x - \frac{\ln(1+x^2)}{2} + C.$$

Answer.

$$\textbf{a.}\ \int \frac{\ln^2 x}{x}\ dx = \frac{\ln^3 x}{3}\ +\ C, \qquad \textbf{b.}\ \int \arctan x\ dx = x\arctan x\ -\ \frac{1}{2}\ \ln(1+x^2)\ +\ C.$$