Quiz # 3

Problem 1. Give the definition of a uniformly continuous function.

Definition. A function f is uniformly continuous on (an interval) I if for every $\varepsilon > 0$ there exists $\delta > 0$ such that for every $x, y \in I$ with $|x - y| < \delta$ one has $|f(x) - f(y)| < \varepsilon$.

Problem 2. Differentiate

$$\int_{x^2}^1 e^t \ dt.$$

Solution.

$$\frac{d}{dx} \int_{x^2}^1 e^t \, dt = -e^{x^2} \, \frac{d}{dx} \, x^2 = -2xe^{x^2}.$$

Problem 3. Find

a. $\log_3 27$ **b.** $\log_4 \frac{1}{16}$ **c.** $\ln(\sqrt{e} x^2) - 2\ln x$

Solution.

a.
$$\log_3 27 = \log_3 3^3 = 3$$
,

b.
$$\log_4 \frac{1}{16} = \log_4 4^{-2} = -2,$$

c. $\ln(\sqrt{e} \ x^2) - 2\ln x = \ln e^{1/2} + \ln x^2 - \ln x^2 = \frac{1}{2}.$