Quiz # 2

Problem 1. Provide the definition of a slant asymptote.

Definition. Let $a, b \in \mathbb{R}$, $a \neq 0$. The line $y = ax + b$ is called a slant asymptote of a function f if

$$\lim_{x\to\infty} (f(x) - ax - b) = 0 \qquad \text{or} \qquad \lim_{x\to-\infty} (f(x) - ax - b) = 0.$$

Problem 2. Find the following limits (if exist).

a. $\lim\limits_{x\to 0} \dfrac{x^3}{\sin x - x}$ **b.** $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{x}{\tan x}$ **c.** $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{\sin(x^2-1)}$.

Solution.

a. Here we initially have an indeterminate form of the type $\frac{0}{0}$ as well as after applying L'Hospital's Rule twice. So we apply it three times. Note that each time, before applying it, we check the conditions.

$$\lim_{x\to 0} \frac{x^3}{\sin x - x} = \lim_{x\to 0} \frac{3x^2}{\cos x - 1} = \lim_{x\to 0} \frac{6x}{-\sin x} = \lim_{x\to 0} \frac{6}{-\cos x} = -6$$

(in the last equality we used that cos is a continuous function and $\cos 0 = 1$).

b. We don't have an indeterminate form here, both functions are continuous, hence

$$\lim_{x\to \frac{\pi}{4}} \frac{x}{\tan x} = \frac{\pi/4}{\tan(\pi/4)} = \frac{\pi}{4}$$

c. Here we have an indeterminate form of the type $\frac{0}{0}$. Applying L'Hospital's Rule we obtain

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sin(x^2-1)} = \lim_{x\to 1} \frac{1/(2\sqrt{x})}{2x\cos(x^2-1)} = \frac{1}{4}$$

(in the last equality we used continuity of corresponding functions). □

Answer.

a. $\lim\limits_{x\to 0} \dfrac{x^3}{\sin x - x} = -6,$ **b.** $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{x}{\tan x} = \dfrac{\pi}{4},$ **c.** $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{\sin(x^2-1)} = \dfrac{1}{4}.$