FINAL EXAM

MATH 118, WINTER 2008

Closed books, closed notes. You are allowed to refer to results that have appeared in class or in homework as long as you cite the result (except when the entire problem was done in class or in homework). Please write your solutions carefully. Provide as many details as possible. Use one examination booklet as scratch paper, and submit the other one. Good luck.

Problem 1. Write down the definitions of

(a) improper integral of form $\int_a^{+\infty} f$
(b) uniformly continuous function

Problem 2.

(a) State and prove the *Mean Value Theorem.*
(b) Prove that $f(x) = \log_a x$ is differentiable when $a > 1$; find the derivative.
(c) Prove that every absolutely convergent series converges.

Problem 3. Let $y = \frac{x^4}{3} - 8x^2$. Find intervals on which f is increasing, decreasing, convex, and concave. Find the maxima, the minima, and the inflection points of the function. Sketch the graph.

Problem 4. Find $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x(1 - \cos x)}$.

Problem 5. Evaluate the following integrals:

$$\int \frac{dx}{x(1 - \ln^2 x)} \qquad \int_1^2 x e^x \, dx \qquad \int_0^{2\pi} \sin^2(x+1)\, dx$$

$$\int_{-\infty}^{+\infty} \sin x \, dx \qquad \int_0^{100} f, \quad \text{where } f(x) = \begin{cases} x & \text{if } x \text{ is an integer} \\ 0 & \text{otherwise} \end{cases}$$

Problem 6. Find the volume of the solid bounded by the paraboloid $z = 4x^2 + y^2$ and the plane $z = 4$.

Problem 7. Determine whether the following series converges or diverges:

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n}\ln n} \qquad \text{bonus: } \sum_{n=2}^{\infty} \frac{\ln n}{n\sqrt{n}}$$

Problem 8. Find the Taylor series for the following functions:

$$\sin x^2 \qquad \text{bonus: } \sin^2 x$$