

11. (a) $f(x) = x^3 - 12x + 1 \Rightarrow f'(x) = 3x^2 - 12 = 3(x+2)(x-2)$.

We don't need to include "3" in the chart to determine the sign of $f'(x)$.

Interval	$x + 2$	$x - 2$	$f'(x)$	f
$x < -2$	-	-	+	increasing on $(-\infty, -2)$
$-2 < x < 2$	+	-	-	decreasing on $(-2, 2)$
$x > 2$	+	+	+	increasing on $(2, \infty)$

So f is increasing on $(-\infty, -2)$ and $(2, \infty)$ and f is decreasing on $(-2, 2)$.

(b) f changes from increasing to decreasing at $x = -2$ and from decreasing to increasing at $x = 2$. Thus, $f(-2) = 17$ is a local maximum value and $f(2) = -15$ is a local minimum value.

(c) $f''(x) = 6x$. $f''(x) > 0 \Leftrightarrow x > 0$ and $f''(x) < 0 \Leftrightarrow x < 0$. Thus, f is concave upward on $(0, \infty)$ and concave downward on $(-\infty, 0)$. There is an inflection point where the concavity changes, at $(0, f(0)) = (0, 1)$.

12. (a) $f(x) = 5 - 3x^2 + x^3 \Rightarrow f'(x) = -6x + 3x^2 = 3x(x-2)$. Thus, $f'(x) > 0 \Leftrightarrow x < 0$ or $x > 2$ and $f'(x) < 0 \Leftrightarrow 0 < x < 2$. So f is increasing on $(-\infty, 0)$ and $(2, \infty)$ and f is decreasing on $(0, 2)$.

(b) f changes from increasing to decreasing at $x = 0$ and from decreasing to increasing at $x = 2$. Thus, $f(0) = 5$ is a local maximum value and $f(2) = 1$ is a local minimum value.

(c) $f''(x) = -6 + 6x = 6(x-1)$. $f''(x) > 0 \Leftrightarrow x > 1$ and $f''(x) < 0 \Leftrightarrow x < 1$. Thus, f is concave upward on $(1, \infty)$ and concave downward on $(-\infty, 1)$. There is an inflection point at $(1, 3)$.

13. (a) $f(x) = x^4 - 2x^2 + 3 \Rightarrow f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x+1)(x-1)$.

Interval	$x + 1$	x	$x - 1$	$f'(x)$	f
$x < -1$	-	-	-	-	decreasing on $(-\infty, -1)$
$-1 < x < 0$	+	-	-	+	increasing on $(-1, 0)$
$0 < x < 1$	+	+	-	-	decreasing on $(0, 1)$
$x > 1$	+	+	+	+	increasing on $(1, \infty)$

So f is increasing on $(-1, 0)$ and $(1, \infty)$ and f is decreasing on $(-\infty, -1)$ and $(0, 1)$.

(b) f changes from increasing to decreasing at $x = 0$ and from decreasing to increasing at $x = -1$ and $x = 1$. Thus, $f(0) = 3$ is a local maximum value and $f(\pm 1) = 2$ are local minimum values.

(c) $f''(x) = 12x^2 - 4 = 12(x^2 - \frac{1}{3}) = 12(x + 1/\sqrt{3})(x - 1/\sqrt{3})$. $f''(x) > 0 \Leftrightarrow x < -1/\sqrt{3}$ or $x > 1/\sqrt{3}$ and $f''(x) < 0 \Leftrightarrow -1/\sqrt{3} < x < 1/\sqrt{3}$. Thus, f is concave upward on $(-\infty, -\sqrt{3}/3)$ and $(\sqrt{3}/3, \infty)$ and concave downward on $(-\sqrt{3}/3, \sqrt{3}/3)$. There are inflection points at $(\pm\sqrt{3}/3, \frac{22}{9})$.

14. (a) $f(x) = \frac{x^2}{x^2 + 3} \Rightarrow f'(x) = \frac{(x^2 + 3)(2x) - x^2(2x)}{(x^2 + 3)^2} = \frac{6x}{(x^2 + 3)^2}$. The denominator is positive so the sign of $f'(x)$ is determined by the sign of x . Thus, $f'(x) > 0 \Leftrightarrow x > 0$ and $f'(x) < 0 \Leftrightarrow x < 0$. So f is increasing on $(0, \infty)$ and f is decreasing on $(-\infty, 0)$.

(b) f changes from decreasing to increasing at $x = 0$. Thus, $f(0) = 0$ is a local minimum value.

(c) $f''(x) =$

$f''(x) >$

and concave

15. (a) $f(x) = x$

$\Leftrightarrow \frac{\pi}{3} <$

increasing

(b) f changes

Thus, $f(\frac{1}{2})$

$f(\frac{7\pi}{3}) =$

(c) $f''(x) =$

upward on

and $(2\pi, \frac{5\pi}{2})$

16. (a) $f(x) = \cos x$

$1 + \sin x$

$\Rightarrow \cos x$

$\Leftrightarrow 0 <$

and $(3\pi/2, 2\pi)$

(b) f changes

$f(\pi/2) =$

(c) $f''(x) =$

$=$

so $f''(x)$

$0 < x <$

on $(0, \frac{\pi}{6})$

17. $f(x) = x^5 -$

First Deri

from positive

positive at x

Second De

local maximu

Preferenc

18. $f(x) = \frac{x}{x^2 + 3}$

First Deri

from positive

positive at x

$$\begin{aligned} \text{(c) } f''(x) &= \frac{(x^2 + 3)^2(6) - 6x \cdot 2(x^2 + 3)(2x)}{[(x^2 + 3)^2]^2} = \frac{6(x^2 + 3)[x^2 + 3 - 4x^2]}{(x^2 + 3)^4} \\ &= \frac{6(3 - 3x^2)}{(x^2 + 3)^3} = \frac{-18(x + 1)(x - 1)}{(x^2 + 3)^3}. \end{aligned}$$

$f''(x) > 0 \Leftrightarrow -1 < x < 1$ and $f''(x) < 0 \Leftrightarrow x < -1$ or $x > 1$. Thus, f is concave upward on $(-1, 1)$ and concave downward on $(-\infty, -1)$ and $(1, \infty)$. There are inflection points at $(\pm 1, \frac{1}{4})$.

15. (a) $f(x) = x - 2 \sin x$ on $(0, 3\pi) \Rightarrow f'(x) = 1 - 2 \cos x$. $f'(x) > 0 \Leftrightarrow 1 - 2 \cos x > 0 \Leftrightarrow \cos x < \frac{1}{2}$
 $\Leftrightarrow \frac{\pi}{3} < x < \frac{5\pi}{3}$ or $\frac{7\pi}{3} < x < 3\pi$. $f'(x) < 0 \Leftrightarrow \cos x > \frac{1}{2} \Leftrightarrow 0 < x < \frac{\pi}{3}$ or $\frac{5\pi}{3} < x < \frac{7\pi}{3}$. So f is increasing on $(\frac{\pi}{3}, \frac{5\pi}{3})$ and $(\frac{7\pi}{3}, 3\pi)$, and f is decreasing on $(0, \frac{\pi}{3})$ and $(\frac{5\pi}{3}, \frac{7\pi}{3})$.

(b) f changes from increasing to decreasing at $x = \frac{5\pi}{3}$, and from decreasing to increasing at $x = \frac{\pi}{3}$ and at $x = \frac{7\pi}{3}$. Thus, $f(\frac{5\pi}{3}) = \frac{5\pi}{3} + \sqrt{3} \approx 6.97$ is a local maximum value and $f(\frac{\pi}{3}) = \frac{\pi}{3} - \sqrt{3} \approx -0.68$ and $f(\frac{7\pi}{3}) = \frac{7\pi}{3} - \sqrt{3} \approx 5.60$ are local minimum values.

(c) $f''(x) = 2 \sin x > 0 \Leftrightarrow 0 < x < \pi$ and $2\pi < x < 3\pi$, $f''(x) < 0 \Leftrightarrow \pi < x < 2\pi$. Thus, f is concave upward on $(0, \pi)$ and $(2\pi, 3\pi)$, and f is concave downward on $(\pi, 2\pi)$. There are inflection points at (π, π) and $(2\pi, 2\pi)$.

16. (a) $f(x) = \cos^2 x - 2 \sin x$, $0 \leq x \leq 2\pi$. $f'(x) = -2 \cos x \sin x - 2 \cos x = -2 \cos x(1 + \sin x)$. Note that $1 + \sin x \geq 0$ [since $\sin x \geq -1$], with equality $\Leftrightarrow \sin x = -1 \Leftrightarrow x = 3\pi/2$ [since $0 \leq x \leq 2\pi$] $\Rightarrow \cos x = 0$. Thus, $f'(x) > 0 \Leftrightarrow \cos x < 0 \Leftrightarrow \pi/2 < x < 3\pi/2$ and $f'(x) < 0 \Leftrightarrow \cos x > 0 \Leftrightarrow 0 < x < \pi/2$ or $3\pi/2 < x < 2\pi$. Thus, f is increasing on $(\pi/2, 3\pi/2)$ and f is decreasing on $(0, \pi/2)$ and $(3\pi/2, 2\pi)$.

(b) f changes from decreasing to increasing at $x = \pi/2$ and from increasing to decreasing at $x = 3\pi/2$. Thus, $f(\pi/2) = -2$ is a local minimum value and $f(3\pi/2) = 2$ is a local maximum value.

(c) $f''(x) = 2 \sin x(1 + \sin x) - 2 \cos^2 x = 2 \sin x + 2 \sin^2 x - 2(1 - \sin^2 x)$
 $= 4 \sin^2 x + 2 \sin x - 2 = 2(2 \sin x - 1)(\sin x + 1)$
 so $f''(x) > 0 \Leftrightarrow \sin x > \frac{1}{2} \Leftrightarrow \frac{\pi}{6} < x < \frac{5\pi}{6}$, and $f''(x) < 0 \Leftrightarrow \sin x < \frac{1}{2}$ and $\sin x \neq -1 \Leftrightarrow 0 < x < \frac{\pi}{6}$ or $\frac{5\pi}{6} < x < \frac{3\pi}{2}$ or $\frac{3\pi}{2} < x < 2\pi$. Thus, f is concave upward on $(\frac{\pi}{6}, \frac{5\pi}{6})$ and concave downward on $(0, \frac{\pi}{6})$, $(\frac{5\pi}{6}, \frac{3\pi}{2})$, and $(\frac{3\pi}{2}, 2\pi)$. There are inflection points at $(\frac{\pi}{6}, -\frac{1}{4})$ and $(\frac{5\pi}{6}, -\frac{1}{4})$.

17. $f(x) = x^5 - 5x + 3 \Rightarrow f'(x) = 5x^4 - 5 = 5(x^2 + 1)(x + 1)(x - 1)$.

First Derivative Test: $f'(x) < 0 \Rightarrow -1 < x < 1$ and $f'(x) > 0 \Rightarrow x > 1$ or $x < -1$. Since f' changes from positive to negative at $x = -1$, $f(-1) = 7$ is a local maximum value; and since f' changes from negative to positive at $x = 1$, $f(1) = -1$ is a local minimum value.

Second Derivative Test: $f''(x) = 20x^3$. $f'(x) = 0 \Leftrightarrow x = \pm 1$. $f''(-1) = -20 < 0 \Rightarrow f(-1) = 7$ is a local maximum value. $f''(1) = 20 > 0 \Rightarrow f(1) = -1$ is a local minimum value.

Preference: For this function, the two tests are equally easy.

18. $f(x) = \frac{x}{x^2 + 4} \Rightarrow f'(x) = \frac{(x^2 + 4) \cdot 1 - x(2x)}{(x^2 + 4)^2} = \frac{4 - x^2}{(x^2 + 4)^2} = \frac{(2 + x)(2 - x)}{(x^2 + 4)^2}$.

First Derivative Test: $f'(x) > 0 \Rightarrow -2 < x < 2$ and $f'(x) < 0 \Rightarrow x > 2$ or $x < -2$. Since f' changes from positive to negative at $x = 2$, $f(2) = \frac{1}{4}$ is a local maximum value; and since f' changes from negative to positive at $x = -2$, $f(-2) = -\frac{1}{4}$ is a local minimum value.