31. $f(x) = 5x^2 + 4x \Rightarrow f'(x) = 10x + 4$. $f'(x) = 0 \Rightarrow x = -\frac{2}{5}$, so $-\frac{2}{5}$ is the only critical number.

32. $f(x) = x^3 + x^2 - x \Rightarrow f'(x) = 3x^2 + 2x - 1$. $f'(x) = 0 \Rightarrow (x+1)(3x-1) = 0 \Rightarrow x = -1, \frac{1}{3}$. These are the only critical numbers.

33. $f(x) = x^3 + 3x^2 - 24x \Rightarrow f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8)$.
$f'(x) = 0 \Rightarrow 3(x+4)(x-2) = 0 \Rightarrow x = -4, 2$. These are the only critical numbers.

34. $f(x) = x^3 + x^2 + x \Rightarrow f'(x) = 3x^2 + 2x + 1$. $f'(x) = 0 \Rightarrow 3x^2 + 2x + 1 = 0 \Rightarrow$
$x = \dfrac{-2 \pm \sqrt{4 - 12}}{6}$. Neither of these is a real number. Thus, there are no critical numbers.

35. $s(t) = 3t^4 + 4t^3 - 6t^2 \Rightarrow s'(t) = 12t^3 + 12t^2 - 12t$. $s'(t) = 0 \Rightarrow 12t(t^2 + t - 1) \Rightarrow t = 0$ or $t^2 + t - 1 = 0$. Using the quadratic formula to solve the latter equation gives us
$t = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \dfrac{-1 \pm \sqrt{5}}{2} \approx 0.618, -1.618$. The three critical numbers are 0, $\dfrac{-1 \pm \sqrt{5}}{2}$.

36. $f(z) = \dfrac{z+1}{z^2 + z + 1} \Rightarrow f'(z) = \dfrac{(z^2 + z + 1)\,1 - (z+1)(2z+1)}{(z^2 + z + 1)^2} = \dfrac{-z^2 - 2z}{(z^2 + z + 1)^2} = 0 \Leftrightarrow$
$z(z+2) = 0 \Rightarrow z = 0, -2$ are the critical numbers. (Note that $z^2 + z + 1 \neq 0$ since the discriminant < 0.)

37. $g(x) = |2x + 3| = \begin{cases} 2x + 3 & \text{if } 2x + 3 \geq 0 \\ -(2x+3) & \text{if } 2x + 3 < 0 \end{cases} \Rightarrow g'(x) = \begin{cases} 2 & \text{if } x > -\frac{3}{2} \\ -2 & \text{if } x < -\frac{3}{2} \end{cases}$
$g'(x)$ is never 0, but $g'(x)$ does not exist for $x = -\frac{3}{2}$, so $-\frac{3}{2}$ is the only critical number.

38. $g(x) = x^{1/3} - x^{-2/3} \Rightarrow g'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3} = \frac{1}{3}x^{-5/3}(x+2) = \dfrac{x+2}{3x^{5/3}}$.
$g'(-2) = 0$ and $g'(0)$ does not exist, but 0 is not in the domain of g, so the only critical number is -2.

39. $g(t) = 5t^{2/3} + t^{5/3} \Rightarrow g'(t) = \frac{10}{3}t^{-1/3} + \frac{5}{3}t^{2/3}$. $g'(0)$ does not exist, so $t = 0$ is a critical number.
$g'(t) = \frac{5}{3}t^{-1/3}(2 + t) = 0 \Leftrightarrow t = -2$, so $t = -2$ is also a critical number.

40. $g(t) = \sqrt{t}\,(1 - t) = t^{1/2} - t^{3/2} \Rightarrow g'(t) = \dfrac{1}{2\sqrt{t}} - \frac{3}{2}\sqrt{t}$. $g'(0)$ does not exist, so $t = 0$ is a critical number.
$0 = g'(t) = \dfrac{1 - 3t}{2\sqrt{t}} \Rightarrow t = \frac{1}{3}$, so $t = \frac{1}{3}$ is also a critical number.

41. $F(x) = x^{4/5}(x-4)^2 \Rightarrow$
$F'(x) = x^{4/5} \cdot 2(x-4) + (x-4)^2 \cdot \frac{4}{5}x^{-1/5} = \frac{1}{5}x^{-1/5}(x-4)\,[5 \cdot x \cdot 2 + (x-4) \cdot 4]$
$= \dfrac{(x-4)(14x-16)}{5x^{1/5}} = \dfrac{2(x-4)(7x-8)}{5x^{1/5}} = 0$ when $x = 4, \frac{8}{7}$; and $F'(0)$ does not exist.
Critical numbers are 0, $\frac{8}{7}$, 4.

42. $G(x) = \sqrt[3]{x^2 - x} \Rightarrow G'(x) = \frac{1}{3}(x^2 - x)^{-2/3}(2x - 1)$. $G'(x)$ does not exist when $x^2 - x = 0$, that is, when $x = 0$ or 1. $G'(x) = 0 \Leftrightarrow 2x - 1 = 0 \Leftrightarrow x = \frac{1}{2}$. So the critical numbers are $x = 0, \frac{1}{2}, 1$.

43. $f(\theta) = 2\cos\theta + \sin^2\theta \Rightarrow f'(\theta) = -2\sin\theta + 2\sin\theta\cos\theta$. $f'(\theta) = 0 \Rightarrow 2\sin\theta(\cos\theta - 1) = 0 \Rightarrow$ $\sin\theta = 0$ or $\cos\theta = 1 \Rightarrow \theta = n\pi$ (n an integer) or $\theta = 2n\pi$. The solutions $\theta = n\pi$ include the solutions $\theta = 2n\pi$, so the critical numbers are $\theta = n\pi$.

44. $g(\theta) = 4\theta - \tan\theta \Rightarrow g'(\theta) = 4 - \sec^2\theta$. $g'(\theta) = 0 \Rightarrow \sec^2\theta = 4 \Rightarrow \sec\theta = \pm 2 \Rightarrow$
$\cos\theta = \pm\frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi$, and $\frac{4\pi}{3} + 2n\pi$ are critical numbers.
Note: The values of θ that make $g'(\theta)$ undefined are not in the domain of g.

45. $f(x) = 3x^2 - 12x + 5$, $[0, 3]$. $f'(x) = 6x - 12 = 0 \Leftrightarrow x = 2$. Applying the Closed Interval Method, we find that $f(0) = 5$, $f(2) = -7$, and $f(3) = -4$. So $f(0) = 5$ is the absolute maximum value and $f(2) = -7$ is the absolute minimum value.

46. $f(x) = x^3 - 3x + 1$, $[0, 3]$. $f'(x) = 3x^2 - 3 = 0 \Leftrightarrow x = \pm 1$, but -1 is not in $[0, 3]$. $f(0) = 1$, $f(1) = -1$, and $f(3) = 19$. So $f(3) = 19$ is the absolute maximum value and $f(1) = -1$ is the absolute minimum value.

47. $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$. $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1) = 0 \Leftrightarrow$ $x = 2, -1$. $f(-2) = -3$, $f(-1) = 8$, $f(2) = -19$, and $f(3) = -8$. So $f(-1) = 8$ is the absolute maximum value and $f(2) = -19$ is the absolute minimum value.

48. $f(x) = x^3 - 6x^2 + 9x + 2$, $[-1, 4]$. $f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3) = 0 \Leftrightarrow$ $x = 1, 3$. $f(-1) = -14$, $f(1) = 6$, $f(3) = 2$, and $f(4) = 6$. So $f(1) = f(4) = 6$ is the absolute maximum value and $f(-1) = -14$ is the absolute minimum value.

49. $f(x) = x^4 - 2x^2 + 3$, $[-2, 3]$. $f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x + 1)(x - 1) = 0 \Leftrightarrow x = -1, 0, 1$. $f(-2) = 11$, $f(-1) = 2$, $f(0) = 3$, $f(1) = 2$, $f(3) = 66$. So $f(3) = 66$ is the absolute maximum value and $f(\pm 1) = 2$ is the absolute minimum value.

50. $f(x) = (x^2 - 1)^3$, $[-1, 2]$. $f'(x) = 3(x^2 - 1)^2(2x) = 6x(x + 1)^2(x - 1)^2 = 0 \Leftrightarrow x = -1, 0, 1$. $f(\pm 1) = 0$, $f(0) = -1$, and $f(2) = 27$. So $f(2) = 27$ is the absolute maximum value and $f(0) = -1$ is the absolute minimum value.

51. $f(x) = \dfrac{x}{x^2 + 1}$, $[0, 2]$. $f'(x) = \dfrac{(x^2 + 1) - x(2x)}{(x^2 + 1)^2} = \dfrac{1 - x^2}{(x^2 + 1)^2} = 0 \Leftrightarrow x = \pm 1$, but -1 is not in $[0, 2]$. $f(0) = 0$, $f(1) = \frac{1}{2}$, $f(2) = \frac{2}{5}$. So $f(1) = \frac{1}{2}$ is the absolute maximum value and $f(0) = 0$ is the absolute minimum value.

52. $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$, $[-4, 4]$. $f'(x) = \dfrac{(x^2 + 4)(2x) - (x^2 - 4)(2x)}{(x^2 + 4)^2} = \dfrac{16x}{(x^2 + 4)^2} = 0 \Leftrightarrow x = 0$. $f(\pm 4) = \frac{12}{20} = \frac{3}{5}$ and $f(0) = -1$. So $f(\pm 4) = \frac{3}{5}$ is the absolute maximum value and $f(0) = -1$ is the absolute minimum value.

53. $f(t) = t\sqrt{4 - t^2}$, $[-1, 2]$.

$$f'(t) = t \cdot \tfrac{1}{2}(4 - t^2)^{-1/2}(-2t) + (4 - t^2)^{1/2} \cdot 1 = \frac{-t^2}{\sqrt{4 - t^2}} + \sqrt{4 - t^2} = \frac{-t^2 + (4 - t^2)}{\sqrt{4 - t^2}} = \frac{4 - 2t^2}{\sqrt{4 - t^2}}.$$

$f'(t) = 0 \Rightarrow 4 - 2t^2 = 0 \Rightarrow t^2 = 2 \Rightarrow t = \pm\sqrt{2}$, but $t = -\sqrt{2}$ is not in the given interval, $[-1, 2]$. $f'(t)$ does not exist if $4 - t^2 = 0 \Rightarrow t = \pm 2$, but -2 is not in the given interval. $f(-1) = -\sqrt{3}$, $f(\sqrt{2}) = 2$, and $f(2) = 0$. So $f(\sqrt{2}) = 2$ is the absolute maximum value and $f(-1) = -\sqrt{3}$ is the absolute minimum value.

54. $f(t) = \sqrt[3]{t}\,(8 - t)$, $[0, 8]$. $f(t) = 8t^{1/3} - t^{4/3} \Rightarrow f'(t) = \frac{8}{3}t^{-2/3} - \frac{4}{3}t^{1/3} = \frac{4}{3}t^{-2/3}(2 - t) = \dfrac{4(2 - t)}{3\sqrt[3]{t^2}}$.

$f'(t) = 0 \Rightarrow t = 2$. $f'(t)$ does not exist if $t = 0$. $f(0) = 0$, $f(2) = 6\sqrt[3]{2} \approx 7.56$, and $f(8) = 0$. So $f(2) = 6\sqrt[3]{2}$ is the absolute maximum value and $f(0) = f(8) = 0$ is the absolute minimum value.

55. $f(x) = \sin x + \cos x$, $[0, \frac{\pi}{3}]$. $f'(x) = \cos x - \sin x = 0 \Leftrightarrow \sin x = \cos x \Rightarrow \dfrac{\sin x}{\cos x} = 1 \Rightarrow$ $\tan x = 1 \Rightarrow x = \frac{\pi}{4}$. $f(0) = 1$, $f(\frac{\pi}{4}) = \sqrt{2} \approx 1.41$, $f(\frac{\pi}{3}) = \frac{\sqrt{3} + 1}{2} \approx 1.37$. So $f(\frac{\pi}{4}) = \sqrt{2}$ is the absolute maximum value and $f(0) = 1$ is the absolute minimum value.