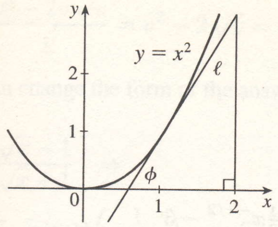
47.



In the right triangle in the diagram, let Δy be the side opposite angle ϕ and Δx the side adjacent angle ϕ. Then the slope of the tangent line ℓ is $m = \Delta y/\Delta x = \tan\phi$. Note that $0 < \phi < \frac{\pi}{2}$. We know (see Exercise 17) that the derivative of $f(x) = x^2$ is $f'(x) = 2x$. So the slope of the tangent to the curve at the point $(1, 1)$ is 2. Thus, ϕ is the angle between 0 and $\frac{\pi}{2}$ whose tangent is 2; that is, $\phi = \tan^{-1} 2 \approx 63°$.

3.3 Differentiation Formulas

1. $f(x) = 186.5$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.

2. $f(x) = \sqrt{30}$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.

3. $f(x) = 5x - 1 \;\Rightarrow\; f'(x) = 5 - 0 = 5$

4. $F(x) = -4x^{10} \;\Rightarrow\; F'(x) = -4(10x^{10-1}) = -40x^9$

5. $f(x) = x^2 + 3x - 4 \;\Rightarrow\; f'(x) = 2x^{2-1} + 3 - 0 = 2x + 3$

6. $g(x) = 5x^8 - 2x^5 + 6 \;\Rightarrow\; g'(x) = 5(8x^{8-1}) - 2(5x^{5-1}) + 0 = 40x^7 - 10x^4$

7. $f(t) = \frac{1}{4}(t^4 + 8) \;\Rightarrow\; f'(t) = \frac{1}{4}(t^4 + 8)' = \frac{1}{4}(4t^{4-1} + 0) = t^3$

8. $f(t) = \frac{1}{2}t^6 - 3t^4 + t \;\Rightarrow\; f'(t) = \frac{1}{2}(6t^5) - 3(4t^3) + 1 = 3t^5 - 12t^3 + 1$

9. $V(r) = \frac{4}{3}\pi r^3 \;\Rightarrow\; V'(r) = \frac{4}{3}\pi(3r^2) = 4\pi r^2$

10. $R(t) = 5t^{-3/5} \;\Rightarrow\; R'(t) = 5\left[-\frac{3}{5}t^{(-3/5)-1}\right] = -3t^{-8/5}$

11. $Y(t) = 6t^{-9} \;\Rightarrow\; Y'(t) = 6(-9)t^{-10} = -54t^{-10}$

12. $R(x) = \dfrac{\sqrt{10}}{x^7} = \sqrt{10}\,x^{-7} \;\Rightarrow\; R'(x) = -7\sqrt{10}\,x^{-8} = -\dfrac{7\sqrt{10}}{x^8}$

13. $F(x) = (\frac{1}{2}x)^5 = (\frac{1}{2})^5 x^5 = \frac{1}{32}x^5 \;\Rightarrow\; F'(x) = \frac{1}{32}(5x^4) = \frac{5}{32}x^4$

14. $f(t) = \sqrt{t} - \dfrac{1}{\sqrt{t}} = t^{1/2} - t^{-1/2} \;\Rightarrow\; f'(t) = \frac{1}{2}t^{-1/2} - \left(-\frac{1}{2}t^{-3/2}\right) = \dfrac{1}{2\sqrt{t}} + \dfrac{1}{2t\sqrt{t}}$

15. $y = x^{-2/5} \;\Rightarrow\; y' = -\frac{2}{5}x^{(-2/5)-1} = -\frac{2}{5}x^{-7/5} = -\dfrac{2}{5x^{7/5}}$

16. $y = \sqrt[3]{x} = x^{1/3} \;\Rightarrow\; y' = \frac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

17. $y = 4\pi^2 \;\Rightarrow\; y' = 0$ since $4\pi^2$ is a constant.

18. $g(u) = \sqrt{2}\,u + \sqrt{3u} = \sqrt{2}\,u + \sqrt{3}\,\sqrt{u} \;\Rightarrow\; g'(u) = \sqrt{2}\,(1) + \sqrt{3}\left(\frac{1}{2}u^{-1/2}\right) = \sqrt{2} + \sqrt{3}/(2\sqrt{u})$

19. $v = t^2 - \dfrac{1}{\sqrt[4]{t^3}} = t^2 - t^{-3/4} \;\Rightarrow\; v' = 2t - (-\frac{3}{4})t^{-7/4} = 2t + \dfrac{3}{4t^{7/4}} = 2t + \dfrac{3}{4t\sqrt[4]{t^3}}$

20. $u = \sqrt[3]{t^2} + 2\sqrt{t^3} = t^{2/3} + 2t^{3/2} \;\Rightarrow\; u' = \frac{2}{3}t^{-1/3} + 2\left(\frac{3}{2}\right)t^{1/2} = \dfrac{2}{3\sqrt[3]{t}} + 3\sqrt{t}$

21. Product Rule: $y = (x^2 + 1)(x^3 + 1) \;\Rightarrow$

$$y' = (x^2 + 1)(3x^2) + (x^3 + 1)(2x) = 3x^4 + 3x^2 + 2x^4 + 2x = 5x^4 + 3x^2 + 2x.$$

Multiplying first: $y = (x^2 + 1)(x^3 + 1) = x^5 + x^3 + x^2 + 1 \;\Rightarrow\; y' = 5x^4 + 3x^2 + 2x$ (equivalent).

31. $y = \dfrac{v^3 - 2v\sqrt{v}}{v} = v^2 - 2\sqrt{v} = v^2 - 2v^{1/2} \Rightarrow y' = 2v - 2\left(\tfrac{1}{2}\right)v^{-1/2} = 2v - v^{-1/2}$.

We can change the form of the answer as follows: $2v - v^{-1/2} = 2v - \dfrac{1}{\sqrt{v}} = \dfrac{2v\sqrt{v} - 1}{\sqrt{v}} = \dfrac{2v^{3/2} - 1}{\sqrt{v}}$

32. $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1} \Rightarrow$

$$y' = \frac{(\sqrt{x}+1)\left(\dfrac{1}{2\sqrt{x}}\right) - (\sqrt{x}-1)\left(\dfrac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+1)^2} = \frac{\dfrac{1}{2} + \dfrac{1}{2\sqrt{x}} - \dfrac{1}{2} + \dfrac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2} = \frac{1}{\sqrt{x}\,(\sqrt{x}+1)^2}$$

33. $y = \dfrac{1}{x^4 + x^2 + 1} \Rightarrow y' = \dfrac{(x^4 + x^2 + 1)(0) - 1\,(4x^3 + 2x)}{(x^4 + x^2 + 1)^2} = -\dfrac{2x(2x^2 + 1)}{(x^4 + x^2 + 1)^2}$

34. $y = x^2 + x + x^{-1} + x^{-2} \Rightarrow y' = 2x + 1 - x^{-2} - 2x^{-3}$

35. $y = ax^2 + bx + c \Rightarrow y' = 2ax + b$

36. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2} = A + Bx^{-1} + Cx^{-2} \Rightarrow y' = -Bx^{-2} - 2Cx^{-3} = -\dfrac{B}{x^2} - 2\dfrac{C}{x^3}$

37. $y = \dfrac{r^2}{1 + \sqrt{r}} \Rightarrow$

$$y' = \frac{(1+\sqrt{r})(2r) - r^2\left(\frac{1}{2}r^{-1/2}\right)}{(1+\sqrt{r})^2} = \frac{2r + 2r^{3/2} - \frac{1}{2}r^{3/2}}{(1+\sqrt{r})^2} = \frac{2r + \frac{3}{2}r^{3/2}}{(1+\sqrt{r})^2} = \frac{\frac{1}{2}r(4 + 3r^{1/2})}{(1+\sqrt{r})^2} = \frac{r(4 + 3\sqrt{r})}{2(1+\sqrt{r})^2}$$

38. $y = \dfrac{cx}{1 + cx} \Rightarrow y' = \dfrac{(1+cx)(c) - (cx)(c)}{(1+cx)^2} = \dfrac{c + c^2x - c^2x}{(1+cx)^2} = \dfrac{c}{(1+cx)^2}$

39. $y = \sqrt[3]{t}\,(t^2 + t + t^{-1}) = t^{1/3}(t^2 + t + t^{-1}) = t^{7/3} + t^{4/3} + t^{-2/3} \Rightarrow$

$y' = \frac{7}{3}t^{4/3} + \frac{4}{3}t^{1/3} - \frac{2}{3}t^{-5/3} = \frac{1}{3}t^{-5/3}(7t^{9/3} + 4t^{6/3} - 2) = (7t^3 + 4t^2 - 2)/(3t^{5/3})$

40. $y = \dfrac{u^6 - 2u^3 + 5}{u^2} = u^4 - 2u + 5u^{-2} \Rightarrow$

$y' = 4u^3 - 2 - 10u^{-3} = 2u^{-3}(2u^6 - u^3 - 5) = 2(2u^6 - u^3 - 5)/u^3$

41. $f(x) = \dfrac{x}{x + c/x} \Rightarrow$

$$f'(x) = \frac{(x + c/x)(1) - x(1 - c/x^2)}{\left(x + \dfrac{c}{x}\right)^2} = \frac{x + c/x - x + c/x}{\left(\dfrac{x^2 + c}{x}\right)^2} = \frac{2c/x}{\dfrac{(x^2+c)^2}{x^2}} \cdot \frac{x^2}{x^2} = \frac{2cx}{(x^2 + c)^2}$$

42. $f(x) = \dfrac{ax + b}{cx + d} \Rightarrow f'(x) = \dfrac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} = \dfrac{acx + ad - acx - bc}{(cx + d)^2} = \dfrac{ad - bc}{(cx + d)^2}$

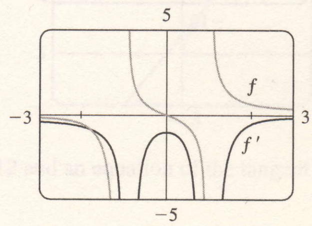
43. $P(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 \Rightarrow$

$P'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + 2a_2x + a_1$

44. $f(x) = \dfrac{x}{x^2 - 1} \Rightarrow$

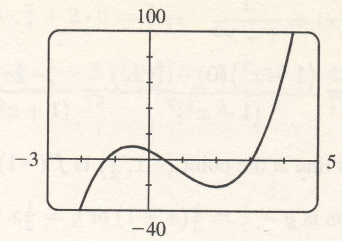
$f'(x) = \dfrac{(x^2 - 1)1 - x(2x)}{(x^2 - 1)^2} = \dfrac{-x^2 - 1}{(x^2 - 1)^2} = -\dfrac{x^2 + 1}{(x^2 - 1)^2}$

Notice that the slopes of all tangents to f are negative and $f'(x) < 0$ always.

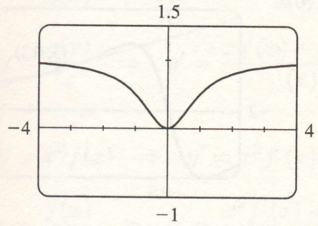


(c) $f(x) = x^4 - 3x^3 - 6x^2 + 7x + 30 \;\Rightarrow$

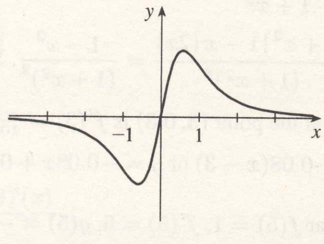
$f'(x) = 4x^3 - 9x^2 - 12x + 7$



50. (a)



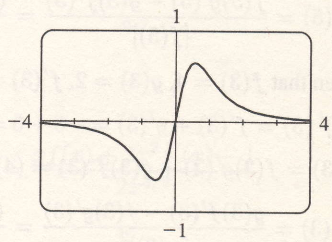
(b)



From the graph in part (a), it appears that g' is zero at $x = 0$. The slopes are negative (so g' is negative) on $(-\infty, 0)$. The slopes are positive (so g' is positive) on $(0, \infty)$.

(c) $g(x) = \dfrac{x^2}{x^2+1} \;\Rightarrow$

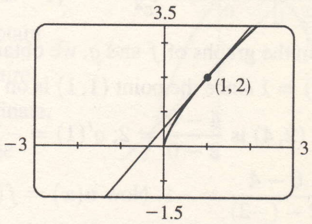
$$g'(x) = \frac{(x^2+1)(2x) - x^2(2x)}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$$



51. $y = \dfrac{2x}{x+1} \;\Rightarrow\; y' = \dfrac{(x+1)(2) - (2x)(1)}{(x+1)^2} = \dfrac{2}{(x+1)^2}$. At $(1,1)$, $y' = \frac{1}{2}$, and an equation of the tangent line is $y - 1 = \frac{1}{2}(x-1)$, or $y = \frac{1}{2}x + \frac{1}{2}$.

52. $y = \dfrac{\sqrt{x}}{x+1} \;\Rightarrow\; y' = \dfrac{(x+1)\left(\dfrac{1}{2\sqrt{x}}\right) - \sqrt{x}\,(1)}{(x+1)^2} = \dfrac{(x+1) - (2x)}{2\sqrt{x}\,(x+1)^2} = \dfrac{1-x}{2\sqrt{x}\,(x+1)^2}$. At $(4, 0.4)$, $y' = \frac{-3}{100} = -0.03$, and an equation of the tangent line is $y - 0.4 = -0.03(x-4)$, or $y = -0.03x + 0.52$.

53. $y = f(x) = x + \sqrt{x} \;\Rightarrow\; f'(x) = 1 + \frac{1}{2}x^{-1/2}$. So the slope of the tangent line at $(1,2)$ is $f'(1) = 1 + \frac{1}{2}(1) = \frac{3}{2}$ and its equation is $y - 2 = \frac{3}{2}(x-1)$ or $y = \frac{3}{2}x + \frac{1}{2}$.



54. $y = (1+2x)^2 = 1 + 4x + 4x^2 \;\Rightarrow\; y' = 4 + 8x$. At $(1,9)$, $y' = 12$ and an equation of the tangent line is $y - 9 = 12(x-1)$ or $y = 12x - 3$.