**9.** (a) $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \dfrac{2n}{3n+1} = \dfrac{2}{3}$, so the *sequence* $\{a_n\}$ is convergent by (12.1.1).

(b) Since $\lim\limits_{n\to\infty} a_n = \frac{2}{3} \neq 0$, the *series* $\sum\limits_{n=1}^{\infty} a_n$ is divergent by the Test for Divergence (7).

**10.** (a) Both $\sum\limits_{i=1}^{n} a_i$ and $\sum\limits_{j=1}^{n} a_j$ represent the sum of the first $n$ terms of the sequence $\{a_n\}$, that is, the $n$th partial sum.

(b) $\sum\limits_{i=1}^{n} a_j = \underbrace{a_j + a_j + \cdots + a_j}_{n \text{ terms}} = na_j$, which, in general, is not the same as $\sum\limits_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$.

**11.** $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$ is a geometric series with first term $a = 3$ and common ratio $r = \frac{2}{3}$. Since $|r| = \frac{2}{3} < 1$, the series converges to $\frac{a}{1-r} = \frac{3}{1-2/3} = \frac{3}{1/3} = 9$.

**12.** $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 + \cdots$ is a geometric series with $r = -2$. Since $|r| = 2 > 1$, the series diverges.

**13.** $-2 + \frac{5}{2} - \frac{25}{8} + \frac{125}{32} - \cdots$ is a geometric series with $a = -2$ and $r = \frac{5/2}{-2} = -\frac{5}{4}$. Since $|r| = \frac{5}{4} > 1$, the series diverges by (4).

**14.** $1 + 0.4 + 0.16 + 0.064 + \cdots$ is a geometric series with ratio 0.4. The series converges to $\frac{a}{1-r} = \frac{1}{1-2/5} = \frac{5}{3}$ since $|r| = \frac{2}{5} < 1$.

**15.** $\sum\limits_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1}$ is a geometric series with $a = 5$ and $r = \frac{2}{3}$. Since $|r| = \frac{2}{3} < 1$, the series converges to $\frac{a}{1-r} = \frac{5}{1-2/3} = \frac{5}{1/3} = 15$.

**16.** $\sum\limits_{n=1}^{\infty} \dfrac{(-6)^{n-1}}{5^{n-1}}$ is a geometric series with $a = 1$ and $r = -\frac{6}{5}$. The series diverges since $|r| = \frac{6}{5} > 1$.

**17.** $\sum\limits_{n=1}^{\infty} \dfrac{(-3)^{n-1}}{4^n} = \dfrac{1}{4}\sum\limits_{n=1}^{\infty}\left(-\dfrac{3}{4}\right)^{n-1}$. The latter series is geometric with $a = 1$ and $r = -\frac{3}{4}$. Since $|r| = \frac{3}{4} < 1$, it converges to $\frac{1}{1-(-3/4)} = \frac{4}{7}$. Thus, the given series converges to $\left(\frac{1}{4}\right)\left(\frac{4}{7}\right) = \frac{1}{7}$.

**18.** $\sum\limits_{n=0}^{\infty} \dfrac{1}{(\sqrt{2})^n}$ is a geometric series with ratio $r = \dfrac{1}{\sqrt{2}}$. Since $|r| = \dfrac{1}{\sqrt{2}} < 1$, the series converges. Its sum is

$$\frac{1}{1-1/\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}-1} = \frac{\sqrt{2}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \sqrt{2}(\sqrt{2}+1) = 2 + \sqrt{2}.$$

**19.** $\sum\limits_{n=0}^{\infty} \dfrac{\pi^n}{3^{n+1}} = \dfrac{1}{3}\sum\limits_{n=0}^{\infty}\left(\dfrac{\pi}{3}\right)^n$ is a geometric series with ratio $r = \dfrac{\pi}{3}$. Since $|r| > 1$, the series diverges.

**20.** $\sum\limits_{n=1}^{\infty} \dfrac{e^n}{3^{n-1}} = 3\sum\limits_{n=1}^{\infty}\left(\dfrac{e}{3}\right)^n$ is a geometric series with first term $3(e/3) = e$ and ratio $r = \dfrac{e}{3}$. Since $|r| < 1$, the series converges. Its sum is $\dfrac{e}{1-e/3} = \dfrac{3e}{3-e}$.

**21.** $\sum\limits_{n=1}^{\infty} \dfrac{n}{n+5}$ diverges since $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \dfrac{n}{n+5} = 1 \neq 0$. [Use (7), the Test for Divergence.]

**22.** $\displaystyle\sum_{n=1}^{\infty}\frac{3}{n}=3\sum_{n=1}^{\infty}\frac{1}{n}$ diverges since each of its partial sums is 3 times the corresponding partial sum of the harmonic series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$, which diverges. [If $\displaystyle\sum_{n=1}^{\infty}\frac{3}{n}$ were to converge, then $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ would also have to converge by Theorem 8(i).] In general, constant multiples of divergent series are divergent.

**23.** Using partial fractions, the partial sums are

$$s_n=\sum_{i=2}^{n}\frac{2}{(i-1)(i+1)}=\sum_{i=2}^{n}\left(\frac{1}{i-1}-\frac{1}{i+1}\right)$$

$$=\left(1-\tfrac{1}{3}\right)+\left(\tfrac{1}{2}-\tfrac{1}{4}\right)+\left(\tfrac{1}{3}-\tfrac{1}{5}\right)+\cdots+\left(\frac{1}{n-3}-\frac{1}{n-1}\right)+\left(\frac{1}{n-2}-\frac{1}{n}\right)$$

This sum is a telescoping series and $s_n=1+\dfrac{1}{2}-\dfrac{1}{n-1}-\dfrac{1}{n}$.

Thus, $\displaystyle\sum_{n=2}^{\infty}\frac{2}{n^2-1}=\lim_{n\to\infty}\left(1+\frac{1}{2}-\frac{1}{n-1}-\frac{1}{n}\right)=\frac{3}{2}$.

**24.** $\displaystyle\sum_{n=1}^{\infty}\frac{(n+1)^2}{n(n+2)}$ diverges by (7), the Test for Divergence, since

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^2+2n+1}{n^2+2n}=\lim_{n\to\infty}\left(1+\frac{1}{n^2+2n}\right)=1\neq 0.$$

**25.** $\displaystyle\sum_{k=2}^{\infty}\frac{k^2}{k^2-1}$ diverges by the Test for Divergence since $\displaystyle\lim_{k\to\infty}a_k=\lim_{k\to\infty}\frac{k^2}{k^2-1}=1\neq 0$.

**26.** Converges. $\displaystyle s_n=\sum_{i=1}^{n}\frac{2}{i^2+4i+3}=\sum_{i=1}^{n}\left(\frac{1}{i+1}-\frac{1}{i+3}\right)$ (using partial fractions). The latter sum is

$$\left(\tfrac{1}{2}-\tfrac{1}{4}\right)+\left(\tfrac{1}{3}-\tfrac{1}{5}\right)+\left(\tfrac{1}{4}-\tfrac{1}{6}\right)+\left(\tfrac{1}{5}-\tfrac{1}{7}\right)+\cdots+\left(\tfrac{1}{n}-\tfrac{1}{n+2}\right)+\left(\tfrac{1}{n+1}-\tfrac{1}{n+3}\right)=\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{n+2}-\tfrac{1}{n+3}$$

(telescoping series). Thus, $\displaystyle\sum_{n=1}^{\infty}\frac{2}{n^2+4n+3}=\lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}\right)=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$.

**27.** Converges. $\displaystyle\sum_{n=1}^{\infty}\frac{3^n+2^n}{6^n}=\sum_{n=1}^{\infty}\left(\frac{3^n}{6^n}+\frac{2^n}{6^n}\right)=\sum_{n=1}^{\infty}\left[\left(\frac{1}{2}\right)^n+\left(\frac{1}{3}\right)^n\right]=\frac{1/2}{1-1/2}+\frac{1/3}{1-1/3}=1+\frac{1}{2}=\frac{3}{2}$

**28.** $\displaystyle\sum_{n=1}^{\infty}\left[(0.8)^{n-1}-(0.3)^n\right]=\sum_{n=1}^{\infty}(0.8)^{n-1}-\sum_{n=1}^{\infty}(0.3)^n$ [difference of two convergent geometric series]

$$=\frac{1}{1-0.8}-\frac{0.3}{1-0.3}=5-\frac{3}{7}=\frac{32}{7}.$$

**29.** $\displaystyle\sum_{n=1}^{\infty}\sqrt[n]{2}=2+\sqrt{2}+\sqrt[3]{2}+\sqrt[4]{2}+\cdots$ diverges by the Test for Divergence since

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\sqrt[n]{2}=\lim_{n\to\infty}2^{1/n}=2^0=1\neq 0.$$