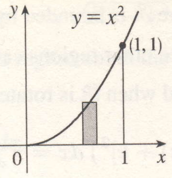
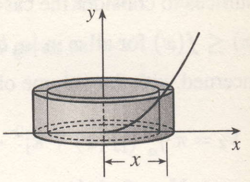
4. $V = \int_0^1 2\pi x \cdot x^2\,dx = 2\pi \int_0^1 x^3\,dx$

$= 2\pi\left[\frac{1}{4}x^4\right]_0^1 = 2\pi \cdot \frac{1}{4} = \frac{\pi}{2}$

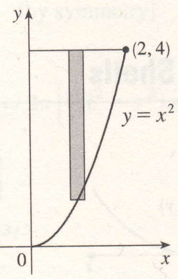


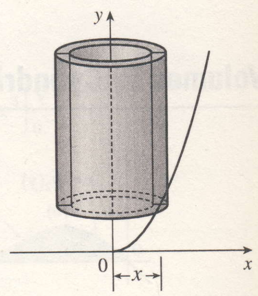


5. $V = \int_0^2 2\pi x\,(4 - x^2)\,dx = 2\pi \int_0^2 (4x - x^3)\,dx$

$= 2\pi\left[2x^2 - \frac{1}{4}x^4\right]_0^2 = 2\pi\,(8 - 4)$

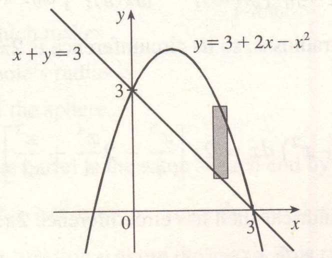
$= 8\pi$

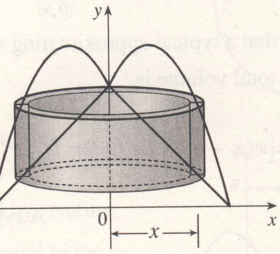




6. $V = 2\pi \int_0^3 \{x[(3 + 2x - x^2) - (3 - x)]\}\,dx = 2\pi \int_0^3 [x(3x - x^2)]\,dx$

$= 2\pi \int_0^3 (3x^2 - x^3)\,dx = 2\pi\left[x^3 - \frac{1}{4}x^4\right]_0^3 = 2\pi\left(27 - \frac{81}{4}\right) = 2\pi\left(\frac{27}{4}\right) = \frac{27\pi}{2}$





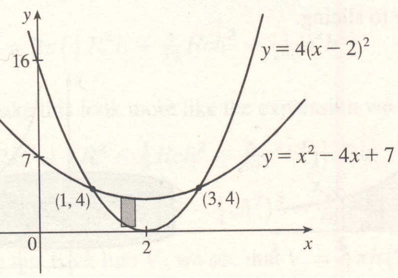
7. The curves intersect when $4(x - 2)^2 = x^2 - 4x + 7 \;\Leftrightarrow\; 4x^2 - 16x + 16 = x^2 - 4x + 7 \;\Leftrightarrow$

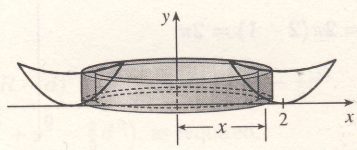
$3x^2 - 12x + 9 = 0 \;\Leftrightarrow\; 3(x^2 - 4x + 3) = 0 \;\Leftrightarrow\; 3(x - 1)(x - 3) = 0$, so $x = 1$ or 3.

$V = 2\pi \int_1^3 \{x[(x^2 - 4x + 7) - 4(x - 2)^2]\}\,dx = 2\pi \int_1^3 [x(x^2 - 4x + 7 - 4x^2 + 16x - 16)]\,dx$

$= 2\pi \int_1^3 [x(-3x^2 + 12x - 9)]\,dx = 2\pi(-3) \int_1^3 (x^3 - 4x^2 + 3x)\,dx = -6\pi\left[\frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{3}{2}x^2\right]_1^3$

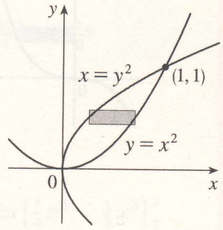
$= -6\pi\left[\left(\frac{81}{4} - 36 + \frac{27}{2}\right) - \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2}\right)\right] = -6\pi\left(20 - 36 + 12 + \frac{4}{3}\right) = -6\pi\left(-\frac{8}{3}\right) = 16\pi$

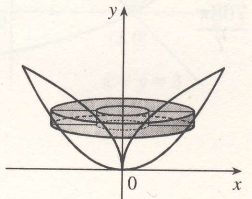




8. By slicing:

$$V = \int_0^1 \pi\left[\left(\sqrt{y}\right)^2 - \left(y^2\right)^2\right] dy = \pi\int_0^1 \left(y - y^4\right) dy = \pi\left[\tfrac{1}{2}y^2 - \tfrac{1}{5}y^5\right]_0^1 = \pi\left(\tfrac{1}{2} - \tfrac{1}{5}\right) = \tfrac{3\pi}{10}$$



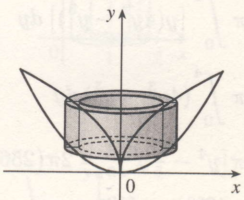


By cylindrical shells:

$$V = \int_0^1 2\pi x\left(\sqrt{x} - x^2\right) dx = 2\pi\int_0^1 \left(x^{3/2} - x^3\right) dx$$

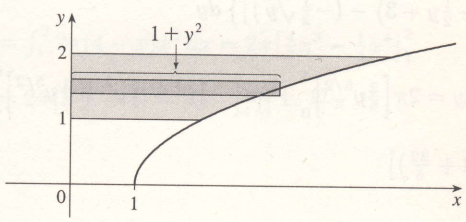
$$= 2\pi\left[\tfrac{2}{5}x^{5/2} - \tfrac{1}{4}x^4\right]_0^1 = 2\pi\left(\tfrac{2}{5} - \tfrac{1}{4}\right)$$

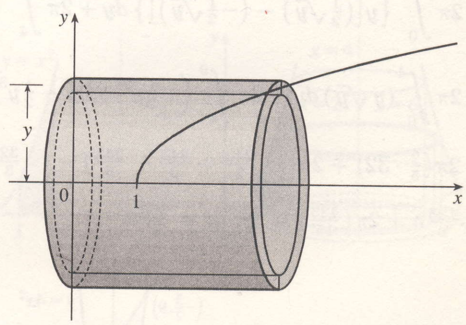
$$= 2\pi\left(\tfrac{3}{20}\right) = \tfrac{3\pi}{10}$$



9. $V = \int_1^2 2\pi y\left(1 + y^2\right) dy = 2\pi\int_1^2 \left(y + y^3\right) dy = 2\pi\left[\tfrac{1}{2}y^2 + \tfrac{1}{4}y^4\right]_1^2$

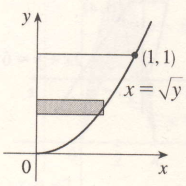
$= 2\pi\left[(2 + 4) - \left(\tfrac{1}{2} + \tfrac{1}{4}\right)\right] = 2\pi\left(\tfrac{21}{4}\right) = \tfrac{21\pi}{2}$

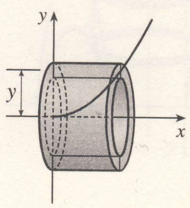




10. $V = \int_0^1 2\pi y\,\sqrt{y}\,dy = 2\pi\int_0^1 y^{3/2}\,dy$

$= 2\pi\left[\tfrac{2}{5}y^{5/2}\right]_0^1 = \tfrac{4\pi}{5}$

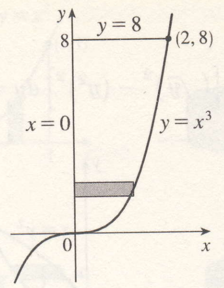


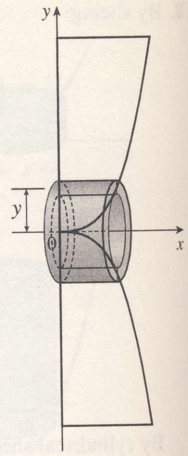


11. $V = 2\pi \int_0^8 [y(\sqrt[3]{y} - 0)]\, dy$

$= 2\pi \int_0^8 y^{4/3}\, dy = 2\pi \left[\frac{3}{7} y^{7/3}\right]_0^8$

$= \frac{6\pi}{7}(8^{7/3}) = \frac{6\pi}{7}(2^7) = \frac{768\pi}{7}$



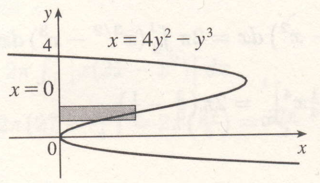


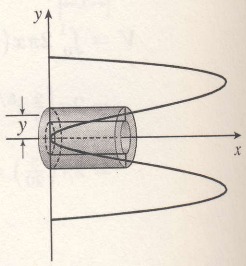
12. $V = 2\pi \int_0^4 [y(4y^2 - y^3)]\, dy$

$= 2\pi \int_0^4 (4y^3 - y^4)\, dy$

$= 2\pi \left[y^4 - \frac{1}{5}y^5\right]_0^4 = 2\pi\left(256 - \frac{1024}{5}\right)$

$= 2\pi\left(\frac{256}{5}\right) = \frac{512\pi}{5}$





13. The curves intersect when $4x^2 = 6 - 2x \;\Leftrightarrow\; 2x^2 + x - 3 = 0 \;\Leftrightarrow\; (2x+3)(x-1) = 0 \;\Leftrightarrow\; x = -\frac{3}{2}$ or 1.

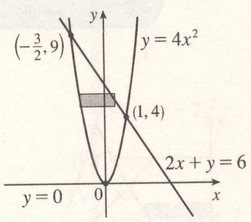
Solving the equations for x gives us $y = 4x^2 \;\Rightarrow\; x = \pm\frac{1}{2}\sqrt{y}$ and $2x + y = 6 \;\Rightarrow\; x = -\frac{1}{2}y + 3$.

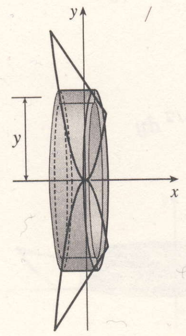
$V = 2\pi \int_0^4 \left\{y\left[\left(\frac{1}{2}\sqrt{y}\right) - \left(-\frac{1}{2}\sqrt{y}\right)\right]\right\} dy + 2\pi \int_4^9 \left\{y\left[\left(-\frac{1}{2}y + 3\right) - \left(-\frac{1}{2}\sqrt{y}\right)\right]\right\} dy$

$= 2\pi \int_0^4 (y\sqrt{y})\, dy + 2\pi \int_4^9 \left(-\frac{1}{2}y^2 + 3y + \frac{1}{2}y^{3/2}\right) dy = 2\pi\left[\frac{2}{5}y^{5/2}\right]_0^4 + 2\pi\left[-\frac{1}{6}y^3 + \frac{3}{2}y^2 + \frac{1}{5}y^{5/2}\right]_4^9$

$= 2\pi\left(\frac{2}{5} \cdot 32\right) + 2\pi\left[\left(-\frac{243}{2} + \frac{243}{2} + \frac{243}{5}\right) - \left(-\frac{32}{3} + 24 + \frac{32}{5}\right)\right]$

$= \frac{128}{5}\pi + 2\pi\left(\frac{433}{15}\right) = \frac{1250}{15}\pi = \frac{250}{3}\pi$



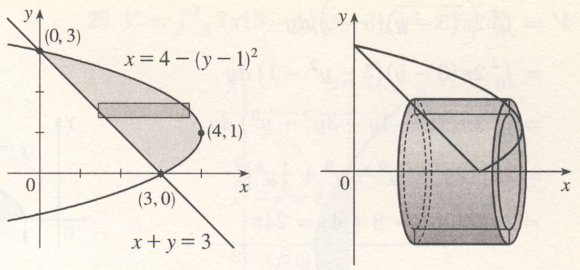


14. $V = \int_0^3 2\pi y\left[4-(y-1)^2-(3-y)\right]dy$

$= 2\pi \int_0^3 y(-y^2+3y)\,dy$

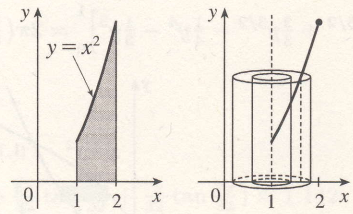
$= 2\pi \int_0^3 (-y^3+3y^2)\,dy = 2\pi\left[-\frac{1}{4}y^4+y^3\right]_0^3$

$= 2\pi\left(-\frac{81}{4}+27\right) = 2\pi\left(\frac{27}{4}\right) = \frac{27\pi}{2}$



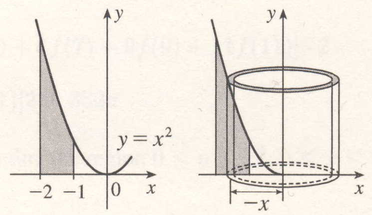
15. $V = \int_1^2 2\pi(x-1)x^2\,dx = 2\pi\left[\frac{1}{4}x^4-\frac{1}{3}x^3\right]_1^2$

$= 2\pi\left[\left(4-\frac{8}{3}\right)-\left(\frac{1}{4}-\frac{1}{3}\right)\right] = \frac{17}{6}\pi$



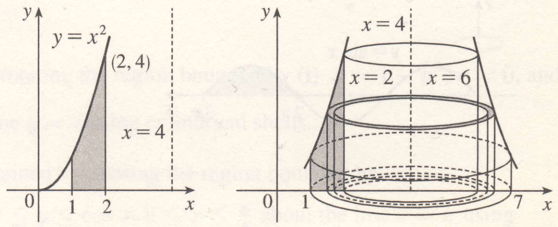
16. $V = \int_{-2}^{-1} 2\pi(-x)\cdot x^2\,dx = 2\pi\left[-\frac{1}{4}x^4\right]_{-2}^{-1}$

$= 2\pi\left[\left(-\frac{1}{4}\right)-(-4)\right] = \frac{15}{2}\pi$



17. $V = \int_1^2 2\pi(4-x)x^2\,dx = 2\pi\left[\frac{4}{3}x^3-\frac{1}{4}x^4\right]_1^2$

$= 2\pi\left[\left(\frac{32}{3}-4\right)-\left(\frac{4}{3}-\frac{1}{4}\right)\right] = \frac{67}{6}\pi$



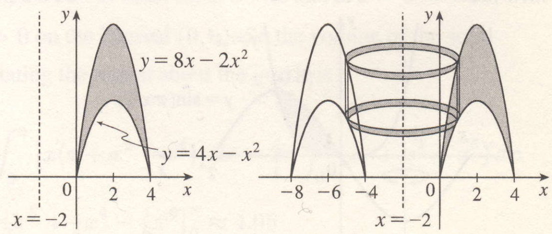
18. $V = \int_0^4 2\pi[x-(-2)][(8x-2x^2)-(4x-x^2)]\,dx$

$= \int_0^4 2\pi(2+x)(4x-x^2)\,dx$

$= 2\pi \int_0^4 (8x+2x^2-x^3)\,dx$

$= 2\pi\left[4x^2+\frac{2}{3}x^3-\frac{1}{4}x^4\right]_0^4$

$= 2\pi\left(64+\frac{128}{3}-64\right) = \frac{256}{3}\pi$



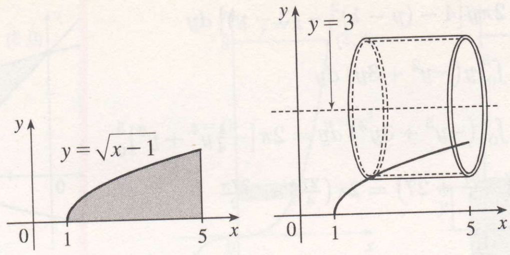
19. $V = \int_0^2 2\pi(3-y)(5-x)dy$

$= \int_0^2 2\pi(3-y)(5-y^2-1)\,dy$

$= \int_0^2 2\pi(12-4y-3y^2+y^3)\,dy$

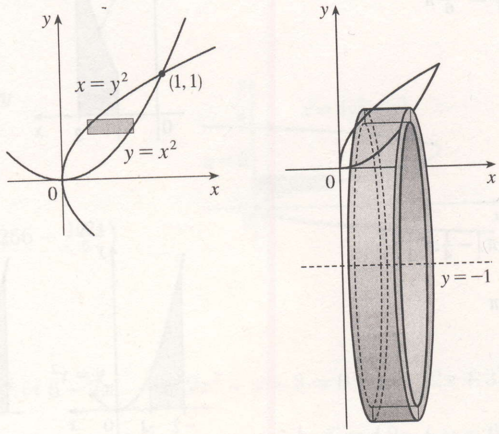
$= 2\pi\left[12y-2y^2-y^3+\frac{1}{4}y^4\right]_0^2$

$= 2\pi(24-8-8+4) = 24\pi$

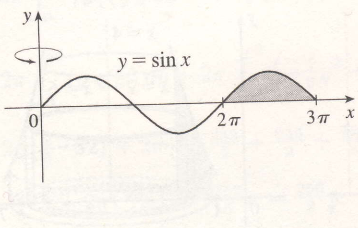


20. $V = \displaystyle\int_0^1 2\pi(y+1)(\sqrt{y}-y^2)\,dy = 2\pi\int_0^1 (y^{3/2}+y^{1/2}-y^3-y^2)\,dy$

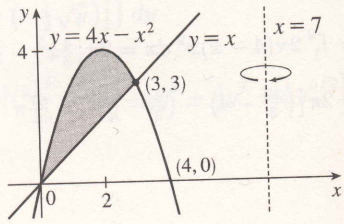
$= 2\pi\left[\frac{2}{5}y^{5/2}+\frac{2}{3}y^{3/2}-\frac{1}{4}y^4-\frac{1}{3}y^3\right]_0^1 = 2\pi\left(\frac{2}{5}+\frac{2}{3}-\frac{1}{4}-\frac{1}{3}\right) = 2\pi\left(\frac{29}{60}\right) = \frac{29\pi}{30}$



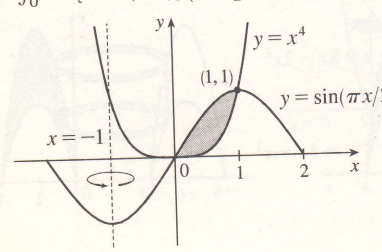
21. $V = \int_{2\pi}^{3\pi} 2\pi x \sin x\,dx$



22. $V = \int_0^3 2\pi(7-x)\left[(4x-x^2)-x\right]dx$



23. $V = \int_0^1 2\pi[x-(-1)](\sin\frac{\pi}{2}x - x^4)\,dx$



24. $V = \displaystyle\int_0^2 2\pi(2-x)\left(\frac{1}{1+x^2}\right)dx$