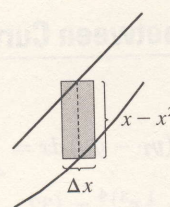
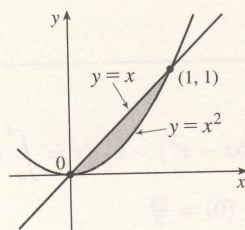
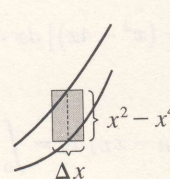
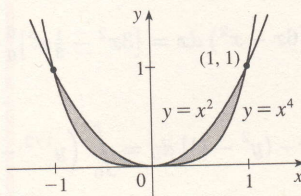


7. The curves intersect when $x = x^2 \Rightarrow x^2 - x = 0 \Leftrightarrow x(x - 1) = 0 \Leftrightarrow x = 0, 1$.

$$\begin{aligned} A &= \int_0^1 (x - x^2) dx \\ &= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^1 \\ &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

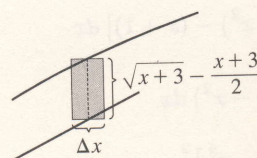
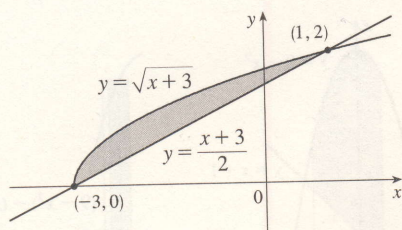


$$\begin{aligned} 8. A &= \int_{-1}^1 (x^2 - x^4) dx \\ &= 2 \int_0^1 (x^2 - x^4) dx \\ &= 2 \left[\frac{1}{3}x^3 - \frac{1}{5}x^5 \right]_0^1 \\ &= 2 \left(\frac{1}{3} - \frac{1}{5} \right) = \frac{4}{15} \end{aligned}$$



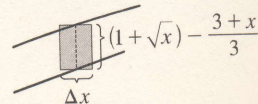
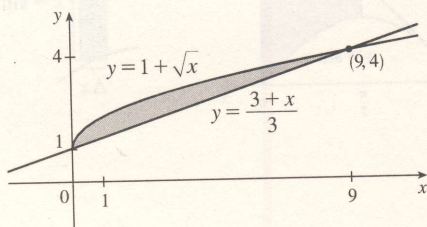
9. First find the points of intersection: $\sqrt{x+3} = \frac{x+3}{2} \Rightarrow (\sqrt{x+3})^2 = \left(\frac{x+3}{2}\right)^2 \Rightarrow x+3 = \frac{1}{4}(x+3)^2$
 $\Rightarrow 4(x+3) - (x+3)^2 = 0 \Rightarrow (x+3)[4 - (x+3)] = 0 \Rightarrow (x+3)(1-x) = 0 \Rightarrow x = -3 \text{ or } 1$. So

$$A = \int_{-3}^1 \left(\sqrt{x+3} - \frac{x+3}{2} \right) dx = \left[\frac{2}{3}(x+3)^{3/2} - \frac{(x+3)^2}{4} \right]_{-3}^1 = \left(\frac{16}{3} - 4 \right) - (0 - 0) = \frac{4}{3}$$

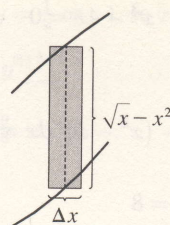
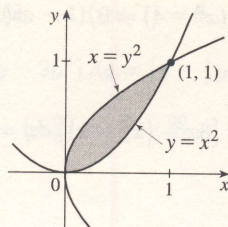


10. $1 + \sqrt{x} = \frac{3+x}{3} = 1 + \frac{x}{3} \Rightarrow \sqrt{x} = \frac{x}{3} \Rightarrow x = \frac{x^2}{9} \Rightarrow 9x - x^2 = 0 \Rightarrow x(9 - x) = 0 \Rightarrow x = 0$
 or 9, so

$$\begin{aligned} A &= \int_0^9 \left[\left(1 + \sqrt{x}\right) - \left(\frac{3+x}{3}\right) \right] dx = \int_0^9 \left[\left(1 + \sqrt{x}\right) - \left(1 + \frac{x}{3}\right) \right] dx \\ &= \int_0^9 \left(\sqrt{x} - \frac{1}{3}x \right) dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{6}x^2 \right]_0^9 = 18 - \frac{27}{2} = \frac{9}{2} \end{aligned}$$

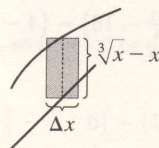
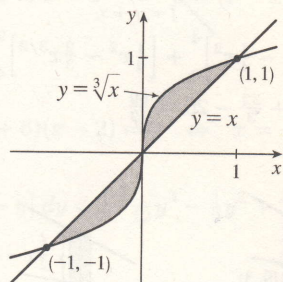


$$\begin{aligned}
 11. A &= \int_0^1 (\sqrt{x} - x^2) dx \\
 &= \left[\frac{2}{3} x^{3/2} - \frac{1}{3} x^3 \right]_0^1 \\
 &= \frac{2}{3} - \frac{1}{3} \\
 &= \frac{1}{3}
 \end{aligned}$$



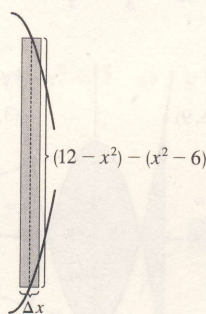
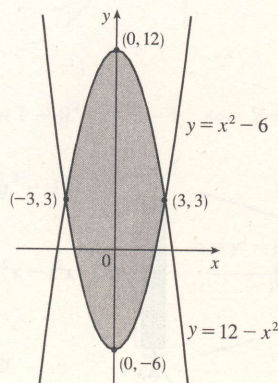
$$\begin{aligned}
 12. x = \sqrt[3]{x} &\Rightarrow x^3 = x \Rightarrow x^3 - x = 0 \Rightarrow x(x^2 - 1) = 0 \Rightarrow x(x+1)(x-1) = 0 \Rightarrow \\
 &x = -1, 0, \text{ or } 1, \text{ so}
 \end{aligned}$$

$$\begin{aligned}
 A &= \int_{-1}^1 |\sqrt[3]{x} - x| dx = \int_{-1}^0 (x - \sqrt[3]{x}) dx + \int_0^1 (\sqrt[3]{x} - x) dx = 2 \int_0^1 (x^{1/3} - x) dx \quad [\text{by symmetry}] \\
 &= 2 \left[\frac{3}{4} x^{4/3} - \frac{1}{2} x^2 \right]_0^1 = 2 \left(\frac{3}{4} - \frac{1}{2} \right) = \frac{1}{2}
 \end{aligned}$$



$$13. 12 - x^2 = x^2 - 6 \Leftrightarrow 2x^2 = 18 \Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm 3, \text{ so}$$

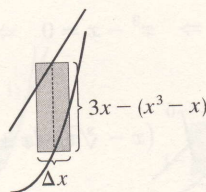
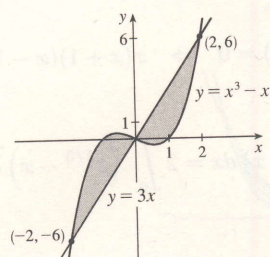
$$\begin{aligned}
 A &= \int_{-3}^3 [(12 - x^2) - (x^2 - 6)] dx = 2 \int_0^3 (18 - 2x^2) dx \quad [\text{by symmetry}] \\
 &= 2 \left[18x - \frac{2}{3} x^3 \right]_0^3 = 2 [(54 - 18) - 0] = 2(36) = 72
 \end{aligned}$$



14. $x^3 - x = 3x \Rightarrow x^3 - 4x = 0 \Rightarrow x(x^2 - 4) = 0 \Rightarrow x(x+2)(x-2) = 0 \Rightarrow x = 0, -2, \text{ or } 2.$

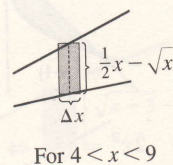
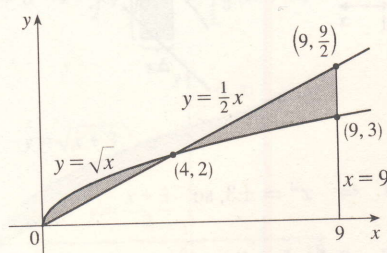
By symmetry,

$$A = \int_{-2}^2 |3x - (x^3 - x)| dx = 2 \int_0^2 [3x - (x^3 - x)] dx = 2 \int_0^2 (4x - x^3) dx = 2 \left[2x^2 - \frac{1}{4}x^4 \right]_0^2 = 2(8 - 4) = 8$$



15. $\frac{1}{2}x = \sqrt{x} \Rightarrow \frac{1}{4}x^2 = x \Rightarrow x^2 - 4x = 0 \Rightarrow x(x - 4) = 0 \Rightarrow x = 0 \text{ or } 4, \text{ so}$

$$A = \int_0^4 (\sqrt{x} - \frac{1}{2}x) dx + \int_4^9 (\frac{1}{2}x - \sqrt{x}) dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{4}x^2 \right]_0^4 + \left[\frac{1}{4}x^2 - \frac{2}{3}x^{3/2} \right]_4^9 = \left[\left(\frac{16}{3} - 4 \right) - 0 \right] + \left[\left(\frac{81}{4} - 18 \right) - \left(4 - \frac{16}{3} \right) \right] = \frac{81}{4} + \frac{32}{3} - 26 = \frac{59}{12}$$



16. $A = \int_{-3}^3 |(8 - x^2) - x^2| dx = 2 \int_0^3 |8 - 2x^2| dx = 2 \int_0^2 (8 - 2x^2) dx + 2 \int_2^3 (2x^2 - 8) dx$

$$= 2 \left[8x - \frac{2}{3}x^3 \right]_0^2 + 2 \left[\frac{2}{3}x^3 - 8x \right]_2^3 = 2 \left[\left(16 - \frac{16}{3} \right) - 0 \right] + 2 \left[(18 - 24) - \left(\frac{16}{3} - 16 \right) \right]$$

$$= 32 - \frac{32}{3} + 20 - \frac{32}{3} = 52 - \frac{64}{3} = \frac{92}{3}$$

