5. $I = \int_1^\infty \frac{1}{(3x+1)^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{(3x+1)^2}\,dx$. Now

$$\int \frac{1}{(3x+1)^2}\,dx = \frac{1}{3}\int \frac{1}{u^2}\,du \quad [u = 3x+1,\ du = 3\,dx]$$
$$= -\frac{1}{3u} + C = -\frac{1}{3(3x+1)} + C,$$

so $I = \lim_{t\to\infty}\left[-\frac{1}{3(3x+1)}\right]_1^t = \lim_{t\to\infty}\left[-\frac{1}{3(3t+1)} + \frac{1}{12}\right] = 0 + \frac{1}{12} = \frac{1}{12}$. Convergent

6. $\int_{-\infty}^0 \frac{1}{2x-5}\,dx = \lim_{t\to-\infty}\int_t^0 \frac{1}{2x-5}\,dx = \lim_{t\to-\infty}\left[\frac12\ln|2x-5|\right]_t^0 = \lim_{t\to\infty}\left[\frac12\ln 5 - \frac12\ln|2t-5|\right] = -\infty$.
Divergent

7. $\int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}}\,dw = \lim_{t\to-\infty}\int_t^{-1} \frac{1}{\sqrt{2-w}}\,dw = \lim_{t\to-\infty}\left[-2\sqrt{2-w}\right]_t^{-1} \quad [u = 2-w,\ du = -dw]$

$= \lim_{t\to-\infty}\left[-2\sqrt{3} + 2\sqrt{2-t}\right] = \infty$. Divergent

8. $\int_0^\infty \frac{x}{(x^2+2)^2}\,dx = \lim_{t\to\infty}\int_0^t \frac{x}{(x^2+2)^2}\,dx = \lim_{t\to\infty}\frac12\left[\frac{-1}{x^2+2}\right]_0^t = \frac12\lim_{t\to\infty}\left(\frac{-1}{t^2+2} + \frac12\right)$

$= \frac12\left(0 + \frac12\right) = \frac14$. Convergent

9. $\int_4^\infty e^{-y/2}\,dy = \lim_{t\to\infty}\int_4^t e^{-y/2}\,dy = \lim_{t\to\infty}\left[-2e^{-y/2}\right]_4^t = \lim_{t\to\infty}(-2e^{-t/2} + 2e^{-2}) = 0 + 2e^{-2} = 2e^{-2}$.
Convergent

10. $\int_{-\infty}^{-1} e^{-2t}\,dt = \lim_{x\to-\infty}\int_x^{-1} e^{-2t}\,dt = \lim_{x\to-\infty}\left[-\frac12 e^{-2t}\right]_x^{-1} = \lim_{x\to-\infty}\left[-\frac12 e^2 + \frac12 e^{-2x}\right] = \infty$. Divergent

11. $\int_{-\infty}^\infty \frac{x\,dx}{1+x^2} = \int_{-\infty}^0 \frac{x\,dx}{1+x^2} + \int_0^\infty \frac{x\,dx}{1+x^2}$ and

$\int_{-\infty}^0 \frac{x\,dx}{1+x^2} = \lim_{t\to-\infty}\left[\frac12\ln(1+x^2)\right]_t^0 = \lim_{t\to-\infty}\left[0 - \frac12\ln(1+t^2)\right] = -\infty$. Divergent

12. $I = \int_{-\infty}^\infty (2-v^4)\,dv = I_1 + I_2 = \int_{-\infty}^0 (2-v^4)\,dv + \int_0^\infty (2-v^4)\,dv$, but
$I_1 = \lim_{t\to-\infty}\left[2v - \frac15 v^5\right]_t^0 = \lim_{t\to-\infty}\left(-2t + \frac15 t^5\right) = -\infty$. Since I_1 is divergent, I is divergent, and there is no need to evaluate I_2. Divergent

13. $\int_{-\infty}^\infty xe^{-x^2}\,dx = \int_{-\infty}^0 xe^{-x^2}\,dx + \int_0^\infty xe^{-x^2}\,dx$.

$\int_{-\infty}^0 xe^{-x^2}\,dx = \lim_{t\to-\infty}\left(-\frac12\right)\left[e^{-x^2}\right]_t^0 = \lim_{t\to-\infty}\left(-\frac12\right)\left(1 - e^{-t^2}\right) = -\frac12\cdot 1 = -\frac12$, and

$\int_0^\infty xe^{-x^2}\,dx = \lim_{t\to\infty}\left(-\frac12\right)\left[e^{-x^2}\right]_0^t = \lim_{t\to\infty}\left(-\frac12\right)\left(e^{-t^2} - 1\right) = -\frac12\cdot(-1) = \frac12$.

Therefore, $\int_{-\infty}^\infty xe^{-x^2}\,dx = -\frac12 + \frac12 = 0$. Convergent

14. $\int_{-\infty}^\infty x^2e^{-x^3}\,dx = \int_{-\infty}^0 x^2e^{-x^3}\,dx + \int_0^\infty x^2e^{-x^3}\,dx$, and

$\int_{-\infty}^0 x^2e^{-x^3}\,dx = \lim_{t\to-\infty}\left[-\frac13 e^{-x^3}\right]_t^0 = -\frac13 + \frac13\left(\lim_{t\to-\infty} e^{-t^3}\right) = \infty$. Divergent

15. $\int_{2\pi}^\infty \sin\theta\,d\theta = \lim_{t\to\infty}\int_{2\pi}^t \sin\theta\,d\theta = \lim_{t\to\infty}\left[-\cos\theta\right]_{2\pi}^t = \lim_{t\to\infty}(-\cos t + 1)$. This limit does not exist, so the integral is divergent. Divergent

16. $\int_0^\infty \cos^2 \alpha \, d\alpha = \lim\limits_{t\to\infty} \int_0^t \frac{1}{2}(1+\cos 2\alpha)\, d\alpha = \lim\limits_{t\to\infty} \left[\frac{1}{2}\alpha + \frac{1}{4}\sin 2\alpha\right]_0^t = \lim\limits_{t\to\infty} \left[\frac{1}{2}t + \frac{1}{4}\sin 2t\right] = \infty$ since $\left|\frac{1}{4}\sin 2t\right| \le \frac{1}{4}$ for all t, but $\frac{1}{2}t \to \infty$ as $t \to \infty$. Divergent

17. $\displaystyle\int_1^\infty \frac{x+1}{x^2+2x}\,dx = \lim_{t\to\infty}\int_1^t \frac{\frac{1}{2}(2x+2)}{x^2+2x}\,dx = \frac{1}{2}\lim_{t\to\infty}\left[\ln(x^2+2x)\right]_1^t = \frac{1}{2}\lim_{t\to\infty}\left[\ln(t^2+2t) - \ln 3\right]$

$= \infty$. Divergent

18. $\displaystyle\int_0^\infty \frac{dz}{z^2+3z+2} = \lim_{t\to\infty}\int_0^t \left[\frac{1}{z+1} - \frac{1}{z+2}\right]dz = \lim_{t\to\infty}\left[\ln\left(\frac{z+1}{z+2}\right)\right]_0^t$

$\displaystyle= \lim_{t\to\infty}\left[\ln\left(\frac{t+1}{t+2}\right) - \ln\left(\frac{1}{2}\right)\right] = \ln 1 + \ln 2 = \ln 2.$ Convergent

19. $\displaystyle\int_0^\infty se^{-5s}\,ds = \lim_{t\to\infty}\int_0^t se^{-5s}\,ds = \lim_{t\to\infty}\left[-\frac{1}{5}se^{-5s} - \frac{1}{25}e^{-5s}\right]_0^t$ [by integration by parts with $u = s$]

$\displaystyle= \lim_{t\to\infty}\left(-\tfrac{1}{5}te^{-5t} - \tfrac{1}{25}e^{-5t} + \tfrac{1}{25}\right) = 0 - 0 + \tfrac{1}{25}$ [by l'Hospital's Rule]

$= \frac{1}{25}$. Convergent

20. $\displaystyle\int_{-\infty}^6 re^{r/3}\,dr = \lim_{t\to-\infty}\int_t^6 re^{r/3}\,dr = \lim_{t\to-\infty}\left[3re^{r/3} - 9e^{r/3}\right]_t^6$ [by integration by parts with $u = r$]

$\displaystyle= \lim_{t\to-\infty}\left(18e^2 - 9e^2 - 3te^{t/3} + 9e^{t/3}\right) = 9e^2 - 0 + 0$ [by l'Hospital's Rule]

$= 9e^2$. Convergent

21. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx = \lim_{t\to\infty}\left[\frac{(\ln x)^2}{2}\right]_1^t$ (by substitution with $u = \ln x$, $du = dx/x$) $\displaystyle= \lim_{t\to\infty}\frac{(\ln t)^2}{2} = \infty.$ Divergent

22. $\int_{-\infty}^\infty e^{-|x|}\,dx = \int_{-\infty}^0 e^x\,dx + \int_0^\infty e^{-x}\,dx$, $\int_{-\infty}^0 e^x\,dx = \lim\limits_{t\to-\infty}\left[e^x\right]_t^0 = \lim\limits_{t\to-\infty}\left(1 - e^t\right) = 1$, and

$\int_0^\infty e^{-x}\,dx = \lim\limits_{t\to\infty}\left[-e^{-x}\right]_0^t = \lim\limits_{t\to\infty}\left(1 - e^{-t}\right) = 1$. Therefore, $\int_{-\infty}^\infty e^{-|x|}\,dx = 1 + 1 = 2$. Convergent

23. $\displaystyle\int_{-\infty}^\infty \frac{x^2}{9+x^6}\,dx = \int_{-\infty}^0 \frac{x^2}{9+x^6}\,dx + \int_0^\infty \frac{x^2}{9+x^6}\,dx = 2\int_0^\infty \frac{x^2}{9+x^6}\,dx$ [since the integrand is even].

Now $\displaystyle\int \frac{x^2\,dx}{9+x^6}\ \left[\begin{array}{c} u = x^3 \\ du = 3x^2\,dx \end{array}\right] = \int \frac{\frac{1}{3}\,du}{9+u^2}\ \left[\begin{array}{c} u = 3v \\ du = 3\,dv \end{array}\right] = \int \frac{\frac{1}{3}(3\,dv)}{9+9v^2} = \frac{1}{9}\int \frac{dv}{1+v^2}$

$\displaystyle= \frac{1}{9}\tan^{-1} v + C = \frac{1}{9}\tan^{-1}\left(\frac{u}{3}\right) + C = \frac{1}{9}\tan^{-1}\left(\frac{x^3}{3}\right) + C,$

so $\displaystyle 2\int_0^\infty \frac{x^2}{9+x^6}\,dx = 2\lim_{t\to\infty}\int_0^t \frac{x^2}{9+x^6}\,dx = 2\lim_{t\to\infty}\left[\frac{1}{9}\tan^{-1}\left(\frac{x^3}{3}\right)\right]_0^t$

$\displaystyle= 2\lim_{t\to\infty}\frac{1}{9}\tan^{-1}\left(\frac{t^3}{3}\right) = \frac{2}{9}\cdot\frac{\pi}{2} = \frac{\pi}{9}.$ Convergent

24. Integrate by parts with $u = \ln x$, $dv = dx/x^3$ $\Rightarrow$ $du = dx/x$, $v = -1/(2x^2)$.

$\displaystyle\int_1^\infty \frac{\ln x}{x^3}\,dx = \lim_{t\to\infty}\int_1^t \frac{\ln x}{x^3}\,dx = \lim_{t\to\infty}\left(\left[-\frac{1}{2x^2}\ln x\right]_1^t + \frac{1}{2}\int_1^t \frac{1}{x^3}\,dx\right)$

$\displaystyle= \lim_{t\to\infty}\left(-\frac{1}{2}\frac{\ln t}{t^2} + 0 - \frac{1}{4t^2} + \frac{1}{4}\right) = \frac{1}{4}$

since $\displaystyle\lim_{t\to\infty}\frac{\ln t}{t^2} \overset{\text{H}}{=} \lim_{t\to\infty}\frac{1/t}{2t} = \lim_{t\to\infty}\frac{1}{2t^2} = 0.$ Convergent

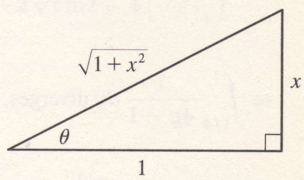
25. Integrate by parts with $u = \ln x$, $dv = dx/x^2$ $\Rightarrow$ $du = dx/x$, $v = -1/x$.

$$\int_1^\infty \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty}\left[-\frac{\ln x}{x} - \frac{1}{x}\right]_1^t = \lim_{t\to\infty}\left(-\frac{\ln t}{t} - \frac{1}{t} + 0 + 1\right)$$
$$= -0 - 0 + 0 + 1 = 1$$

since $\lim\limits_{t\to\infty}\dfrac{\ln t}{t} \overset{\text{H}}{=} \lim\limits_{t\to\infty}\dfrac{1/t}{1} = 0.$ Convergent

26. $\displaystyle\int_0^\infty \frac{x\arctan x}{(1+x^2)^2}\,dx = \lim_{t\to\infty}\int_0^t \frac{x\arctan x}{(1+x^2)^2}\,dx$. Let $u = \arctan x$, $dv = \dfrac{x\,dx}{(1+x^2)^2}$. Then $du = \dfrac{dx}{1+x^2}$, $v = \dfrac{1}{2}\displaystyle\int\frac{2x\,dx}{(1+x^2)^2} = \frac{-1/2}{1+x^2}$, and

$$\int \frac{x\arctan x}{(1+x^2)^2}\,dx = -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{2}\int\frac{dx}{(1+x^2)^2} \qquad \left[\begin{matrix} x = \tan\theta, \\ dx = \sec^2\theta\,d\theta \end{matrix}\right]$$
$$= -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{2}\int\frac{\sec^2\theta\,d\theta}{(\sec^2\theta)^2}$$
$$= -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{2}\int\cos^2\theta\,d\theta$$
$$= -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{\theta}{4} + \frac{\sin\theta\cos\theta}{4} + C$$
$$= -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{4}\arctan x + \frac{1}{4}\frac{x}{1+x^2} + C$$



It follows that

$$\int_0^\infty \frac{x\arctan x}{(1+x^2)^2}\,dx = \lim_{t\to\infty}\left[-\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{4}\arctan x + \frac{1}{4}\frac{x}{1+x^2}\right]_0^t$$
$$= \lim_{t\to\infty}\left(-\frac{1}{2}\frac{\arctan t}{1+t^2} + \frac{1}{4}\arctan t + \frac{1}{4}\frac{t}{1+t^2}\right) = 0 + \frac{1}{4}\cdot\frac{\pi}{2} + 0 = \frac{\pi}{8}.$$

Convergent.

27. There is an infinite discontinuity at the left endpoint of $[0, 3]$.

$$\int_0^3 \frac{dx}{\sqrt{x}} = \lim_{t\to 0^+}\int_t^3 \frac{dx}{\sqrt{x}} = \lim_{t\to 0^+}\left[2\sqrt{x}\right]_t^3 = \lim_{t\to 0^+}\left(2\sqrt{3} - 2\sqrt{t}\right) = 2\sqrt{3}.\quad \text{Convergent}$$

28. There is an infinite discontinuity at the left endpoint of $[0, 3]$.

$$\int_0^3 \frac{dx}{x\sqrt{x}} = \lim_{t\to 0^+}\int_t^3 \frac{dx}{x^{3/2}} = \lim_{t\to 0^+}\left[\frac{-2}{\sqrt{x}}\right]_t^3 = \frac{-2}{\sqrt{3}} + \lim_{t\to 0^+}\frac{2}{\sqrt{t}} = \infty.\quad \text{Divergent}$$

29. There is an infinite discontinuity at the right endpoint of $[-1, 0]$.

$$\int_{-1}^0 \frac{dx}{x^2} = \lim_{t\to 0^-}\int_{-1}^t \frac{dx}{x^2} = \lim_{t\to 0^-}\left[\frac{-1}{x}\right]_{-1}^t = \lim_{t\to 0^-}\left[-\frac{1}{t} + \frac{1}{-1}\right] = \infty.\quad \text{Divergent}$$

30. $\displaystyle\int_1^9 \frac{dx}{\sqrt[3]{x-9}} = \lim_{t\to 9^-}\int_1^t \frac{dx}{\sqrt[3]{x-9}} = \lim_{t\to 9^-}\left[\tfrac{3}{2}(x-9)^{2/3}\right]_1^t = \lim_{t\to 9^-}\left[\tfrac{3}{2}(t-9)^{2/3} - \tfrac{3}{2}(4)\right] = 0 - 6 = -6.$
Convergent

31. $\displaystyle\int_{-2}^3 \frac{dx}{x^4} = \int_{-2}^0 \frac{dx}{x^4} + \int_0^3 \frac{dx}{x^4}$, but $\displaystyle\int_{-2}^0 \frac{dx}{x^4} = \lim_{t\to 0^-}\left[-\frac{x^{-3}}{3}\right]_{-2}^t = \lim_{t\to 0^-}\left[-\frac{1}{3t^3} - \frac{1}{24}\right] = \infty.$ Divergent

32. $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1^-}\int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1^-}\left[\sin^{-1}x\right]_0^t = \lim_{t\to 1^-}\sin^{-1}t = \frac{\pi}{2}.$ Convergent

33. There is an infinite discontinuity at $x = 1$. $\int_0^{33}(x-1)^{-1/5}\,dx = \int_0^1 (x-1)^{-1/5}\,dx + \int_1^{33}(x-1)^{-1/5}\,dx$. Here

$\int_0^1 (x-1)^{-1/5}\,dx = \lim\limits_{t\to1^-}\int_0^t (x-1)^{-1/5}\,dx = \lim\limits_{t\to1^-}\left[\frac{5}{4}(x-1)^{4/5}\right]_0^t = \lim\limits_{t\to1^-}\left[\frac{5}{4}(t-1)^{4/5}-\frac{5}{4}\right] = -\frac{5}{4}$ and

$\int_1^{33}(x-1)^{-1/5}\,dx = \lim\limits_{t\to1^+}\int_t^{33}(x-1)^{-1/5}\,dx = \lim\limits_{t\to1^+}\left[\frac{5}{4}(x-1)^{4/5}\right]_t^{33} = \lim\limits_{t\to1^+}\left[\frac{5}{4}\cdot 16-\frac{5}{4}(t-1)^{4/5}\right] = 20.$

Thus, $\int_0^{33}(x-1)^{-1/5}\,dx = -\frac{5}{4}+20 = \frac{75}{4}$. Convergent

34. $f(y) = 1/(4y-1)$ has an infinite discontinuity at $y = \frac{1}{4}$.

$$\int_{1/4}^1 \frac{1}{4y-1}\,dy = \lim_{t\to(1/4)^+}\int_t^1 \frac{1}{4y-1}\,dy = \lim_{t\to(1/4)^+}\left[\tfrac{1}{4}\ln|4y-1|\right]_t^1$$
$$= \lim_{t\to(1/4)^+}\left[\tfrac{1}{4}\ln 3-\tfrac{1}{4}\ln(4t-1)\right] = \infty$$

so $\int_{1/4}^1 \frac{1}{4y-1}\,dy$ diverges, and hence, $\int_0^1 \frac{1}{4y-1}\,dy$ diverges. Divergent

35. $\int_0^\pi \sec x\,dx = \int_0^{\pi/2}\sec x\,dx + \int_{\pi/2}^\pi \sec x\,dx$. $\int_0^{\pi/2}\sec x\,dx = \lim\limits_{t\to\pi/2^-}\int_0^t \sec x\,dx$

$$= \lim_{t\to\pi/2^-}\Big[\ln|\sec x+\tan x|\Big]_0^t = \lim_{t\to\pi/2^-}\ln|\sec t+\tan t| = \infty.$$ Divergent

36. $\int_0^4 \frac{dx}{x^2+x-6} = \int_0^4 \frac{dx}{(x+3)(x-2)} = \int_0^2 \frac{dx}{(x-2)(x+3)} + \int_2^4 \frac{dx}{(x-2)(x+3)}$, and

$$\int_0^2 \frac{dx}{(x-2)(x+3)} = \lim_{t\to2^-}\int_0^t\left[\frac{1/5}{x-2}-\frac{1/5}{x+3}\right]dx \quad \text{[partial fractions]} \quad = \lim_{t\to2^-}\left[\frac{1}{5}\ln\left|\frac{x-2}{x+3}\right|\right]_0^t$$
$$= \lim_{t\to2^-}\frac{1}{5}\left[\ln\left|\frac{t-2}{t+3}\right|-\ln\frac{2}{3}\right] = -\infty.$$ Divergent

37. There is an infinite discontinuity at $x = 0$. $\int_{-1}^1 \frac{e^x}{e^x-1}\,dx = \int_{-1}^0 \frac{e^x}{e^x-1}\,dx + \int_0^1 \frac{e^x}{e^x-1}\,dx.$

$$\int_{-1}^0 \frac{e^x}{e^x-1}\,dx = \lim_{t\to0^-}\int_{-1}^t \frac{e^x}{e^x-1}\,dx = \lim_{t\to0^-}\Big[\ln|e^x-1|\Big]_{-1}^t = \lim_{t\to0^-}\Big[\ln|e^t-1|-\ln|e^{-1}-1|\Big] = -\infty,$$

so $\int_{-1}^1 \frac{e^x}{e^x-1}\,dx$ is divergent. The integral $\int_0^1 \frac{e^x}{e^x-1}\,dx$ also diverges since

$$\int_0^1 \frac{e^x}{e^x-1}\,dx = \lim_{t\to0^+}\int_t^1 \frac{e^x}{e^x-1}\,dx = \lim_{t\to0^+}\Big[\ln|e^x-1|\Big]_t^1 = \lim_{t\to0^+}\Big[\ln|e-1|-\ln|e^t-1|\Big] = \infty.$$

Divergent

38. $\int_0^2 \frac{x-3}{2x-3}\,dx = \int_0^{3/2}\frac{x-3}{2x-3}\,dx + \int_{3/2}^2 \frac{x-3}{2x-3}\,dx$ and

$$\int \frac{x-3}{2x-3}\,dx = \frac{1}{2}\int\frac{2x-6}{2x-3}\,dx = \frac{1}{2}\int\left[1-\frac{3}{2x-3}\right]dx = \tfrac{1}{2}x-\tfrac{3}{4}\ln|2x-3|+C,$$ so

$$\int_0^{3/2}\frac{x-3}{2x-3}\,dx = \lim_{t\to3/2^-}\tfrac{1}{4}\Big[2x-3\ln|2x-3|\Big]_0^t = \infty.$$ Divergent