20. By Definition 2, $\lim_{h\to 0}\dfrac{\sqrt[4]{16+h}-2}{h} = f'(16)$, where $f(x)=\sqrt[4]{x}$ and $a=16$.

Or: By Definition 2, $\lim_{h\to 0}\dfrac{\sqrt[4]{16+h}-2}{h} = f'(0)$, where $f(x)=\sqrt[4]{16+x}$ and $a=0$.

21. By Equation 3, $\lim_{x\to 5}\dfrac{2^x-32}{x-5} = f'(5)$, where $f(x)=2^x$ and $a=5$.

22. By Equation 3, $\lim_{x\to \pi/4}\dfrac{\tan x-1}{x-\pi/4} = f'(\pi/4)$, where $f(x)=\tan x$ and $a=\pi/4$.

23. By Definition 2, $\lim_{h\to 0}\dfrac{\cos(\pi+h)+1}{h} = f'(\pi)$, where $f(x)=\cos x$ and $a=\pi$.

Or: By Definition 2, $\lim_{h\to 0}\dfrac{\cos(\pi+h)+1}{h} = f'(0)$, where $f(x)=\cos(\pi+x)$ and $a=0$.

24. By Equation 3, $\lim_{t\to 1}\dfrac{t^4+t-2}{t-1} = f'(1)$, where $f(t)=t^4+t$ and $a=1$.

25. $$v(2)=f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{[(2+h)^2-6(2+h)-5]-[2^2-6(2)-5]}{h}$$
$$=\lim_{h\to 0}\frac{(4+4h+h^2-12-6h-5)-(-13)}{h}=\lim_{h\to 0}\frac{h^2-2h}{h}=\lim_{h\to 0}(h-2)=-2\text{ m/s}$$

26. $$v(2)=f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$
$$=\lim_{h\to 0}\frac{[2(2+h)^3-(2+h)+1]-[2(2)^3-2+1]}{h}$$
$$=\lim_{h\to 0}\frac{(2h^3+12h^2+24h+16-2-h+1)-15}{h}$$
$$=\lim_{h\to 0}\frac{2h^3+12h^2+23h}{h}=\lim_{h\to 0}(2h^2+12h+23)=23\text{ m/s}$$

27. (a) $f'(x)$ is the rate of change of the production cost with respect to the number of ounces of gold produced. Its units are dollars per ounce.

(b) After 800 ounces of gold have been produced, the rate at which the production cost is increasing is \$17/ounce. So the cost of producing the 800th (or 801st) ounce is about \$17.

(c) In the short term, the values of $f'(x)$ will decrease because more efficient use is made of start-up costs as x increases. But eventually $f'(x)$ might increase due to large-scale operations.

28. (a) $f'(5)$ is the rate of growth of the bacteria population when $t=5$ hours. Its units are bacteria per hour.

(b) With unlimited space and nutrients, f' should increase as t increases; so $f'(5)<f'(10)$. If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.

29. (a) $f'(v)$ is the rate at which the fuel consumption is changing with respect to the speed. Its units are $(\text{gal/h})/(\text{mi/h})$.

(b) The fuel consumption is decreasing by $0.05\ (\text{gal/h})/(\text{mi/h})$ as the car's speed reaches 20 mi/h. So if you increase your speed to 21 mi/h, you could expect to decrease your fuel consumption by about $0.05\ (\text{gal/h})/(\text{mi/h})$.

30. (a) $f'(8)$ is the rate of change of the quantity of coffee sold with respect to the price per pound when the price is \$8 per pound. The units for $f'(8)$ are pounds/(dollars/pound).

(b) $f'(8)$ is negative since the quantity of coffee sold will decrease as the price charged for it increases. People are generally less willing to buy a product when its price increases.

31. $T'(10)$ is the rate at which the temperature is changing at 10:00 A.M. To estimate the value of $T'(10)$, we will average the difference quotients obtained using the times $t = 8$ and $t = 12$. Let

$$A = \frac{T(8) - T(10)}{8 - 10} = \frac{72 - 81}{-2} = 4.5 \text{ and } B = \frac{T(12) - T(10)}{12 - 10} = \frac{88 - 81}{2} = 3.5. \text{ Then}$$

$$T'(10) = \lim_{t \to 10} \frac{T(t) - T(10)}{t - 10} \approx \frac{A + B}{2} = \frac{4.5 + 3.5}{2} = 4^\circ\text{F/h}.$$

32. For 1910: We will average the difference quotients obtained using the years 1900 and 1920.

$$\text{Let } A = \frac{E(1900) - E(1910)}{1900 - 1910} = \frac{48.3 - 51.1}{-10} = 0.28 \text{ and}$$

$$B = \frac{E(1920) - E(1910)}{1920 - 1910} = \frac{55.2 - 51.1}{10} = 0.41. \text{ Then}$$

$$E'(1910) = \lim_{t \to 1910} \frac{E(t) - E(1910)}{t - 1910} \approx \frac{A + B}{2} = 0.345.$$ This means that life expectancy at birth was increasing at about 0.345 year/year in 1910.

For 1950: Using data for 1940 and 1960 in a similar fashion, we obtain $E'(1950) \approx [0.31 + 0.10]/2 = 0.205$. So life expectancy at birth was increasing at about 0.205 year/year in 1950.

33. (a) $S'(T)$ is the rate at which the oxygen solubility changes with respect to the water temperature. Its units are (mg/L)/°C.

(b) For $T = 16^\circ$C, it appears that the tangent line to the curve goes through the points $(0, 14)$ and $(32, 6)$. So $S'(16) \approx \frac{6 - 14}{32 - 0} = -\frac{8}{32} = -0.25$ (mg/L)/°C. This means that as the temperature increases past 16°C, the oxygen solubility is decreasing at a rate of 0.25 (mg/L)/°C.

34. (a) $S'(T)$ is the rate of change of the maximum sustainable speed of Coho salmon with respect to the temperature. Its units are (cm/s)/°C.

(b) For $T = 15$ °C, it appears the tangent line to the curve goes through the points $(10, 25)$ and $(20, 32)$. So $S'(15) \approx \frac{32 - 25}{20 - 10} = 0.7$ (cm/s)/°C. This tells us that at $T = 15$ °C, the maximum sustainable speed of Coho salmon is changing at a rate of 0.7 (cm/s)/°C. In a similar fashion for $T = 25$ °C, we can use the points $(20, 35)$ and $(25, 25)$ to obtain $S'(25) \approx \frac{25 - 35}{25 - 20} = -2$ (cm/s)/°C. As it gets warmer than 20 °C, the maximum sustainable speed decreases rapidly.

35. Since $f(x) = x\sin(1/x)$ when $x \neq 0$ and $f(0) = 0$, we have

$$f'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \to 0} \frac{h\sin(1/h) - 0}{h} = \lim_{h \to 0} \sin(1/h).$$ This limit does not exist since $\sin(1/h)$ takes the values -1 and 1 on any interval containing 0. (Compare with Example 4 in Section 2.2.)

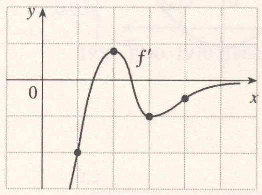
36. Since $f(x) = x^2 \sin(1/x)$ when $x \neq 0$ and $f(0) = 0$, we have

$f'(0) = \lim\limits_{h \to 0} \dfrac{f(0+h) - f(0)}{h} = \lim\limits_{h \to 0} \dfrac{h^2 \sin(1/h) - 0}{h} = \lim\limits_{h \to 0} h \sin(1/h)$. Since $-1 \leq \sin \dfrac{1}{h} \leq 1$, we have

$-|h| \leq |h| \sin \dfrac{1}{h} \leq |h| \quad \Rightarrow \quad -|h| \leq h \sin \dfrac{1}{h} \leq |h|$. Because $\lim\limits_{h \to 0} (-|h|) = 0$ and $\lim\limits_{h \to 0} |h| = 0$, we know that

$\lim\limits_{h \to 0} \left(h \sin \dfrac{1}{h} \right) = 0$ by the Squeeze Theorem. Thus, $f'(0) = 0$.

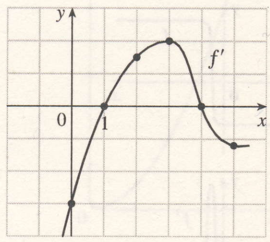
3.2 The Derivative as a Function



1. *Note:* Your answers may vary depending on your estimates. By estimating the slopes of tangent lines on the graph of f, it appears that

(a) $f'(1) \approx -2$ (b) $f'(2) \approx 0.8$

(c) $f'(3) \approx -1$ (d) $f'(4) \approx -0.5$

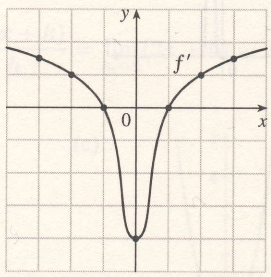


2. *Note:* Your answers may vary depending on your estimates. By estimating the slopes of tangent lines on the graph of f, it appears that

(a) $f'(0) \approx -3$ (b) $f'(1) \approx 0$

(c) $f'(2) \approx 1.5$ (d) $f'(3) \approx 2$

(e) $f'(4) \approx 0$ (f) $f'(5) \approx -1.2$



3. It appears that f is an odd function, so f' will be an even function—that is, $f'(-a) = f'(a)$.

(a) $f'(-3) \approx 1.5$ (b) $f'(-2) \approx 1$

(c) $f'(-1) \approx 0$ (d) $f'(0) \approx -4$

(e) $f'(1) \approx 0$ (f) $f'(2) \approx 1$

(g) $f'(3) \approx 1.5$

4. (a)$'$ = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.

(b)$'$ = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

(c)$'$ = I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

(d)$'$ = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.