14. Let $u = 5t + 4$. Then $du = 5\,dt$ and $dt = \frac{1}{5}\,du$, so

$$\int \frac{1}{(5t+4)^{2.7}}\,dt = \int u^{-2.7}\left(\tfrac{1}{5}\,du\right) = \frac{1}{5}\cdot\frac{1}{-1.7}u^{-1.7} + C = \frac{-1}{8.5}u^{-1.7} + C = \frac{-2}{17(5t+4)^{1.7}} + C.$$

15. Let $u = 4 - t$. Then $du = -dt$ and $dt = -du$, so

$$\int \sqrt{4-t}\,dt = \int u^{1/2}\,(-du) = -\tfrac{2}{3}u^{3/2} + C = -\tfrac{2}{3}(4-t)^{3/2} + C.$$

16. Let $u = 2y^4 - 1$. Then $du = 8y^3\,dy$ and $y^3\,dy = \frac{1}{8}\,du$, so

$$\int y^3\sqrt{2y^4-1}\,dy = \int u^{1/2}\left(\tfrac{1}{8}\,du\right) = \tfrac{1}{8}\cdot\tfrac{2}{3}u^{3/2} + C = \tfrac{1}{12}(2y^4-1)^{3/2} + C.$$

17. Let $u = \pi t$. Then $du = \pi\,dt$ and $dt = \frac{1}{\pi}\,du$, so

$$\int \sin \pi t\,dt = \int \sin u\left(\tfrac{1}{\pi}\,du\right) = \tfrac{1}{\pi}(-\cos u) + C = -\tfrac{1}{\pi}\cos \pi t + C.$$

18. Let $u = 2\theta$. Then $du = 2\,d\theta$ and $d\theta = \frac{1}{2}\,du$, so

$$\int \sec 2\theta\,\tan 2\theta\,d\theta = \int \sec u \tan u\left(\tfrac{1}{2}\,du\right) = \tfrac{1}{2}\sec u + C = \tfrac{1}{2}\sec 2\theta + C.$$

19. Let $u = \sqrt{t}$. Then $du = \dfrac{dt}{2\sqrt{t}}$ and $\dfrac{1}{\sqrt{t}}\,dt = 2\,du$, so

$$\int \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt = \int \cos u\,(2\,du) = 2\sin u + C = 2\sin\sqrt{t} + C.$$

20. Let $u = 1 + x^{3/2}$. Then $du = \frac{3}{2}x^{1/2}\,dx$ and $\sqrt{x}\,dx = \frac{2}{3}\,du$, so

$$\int \sqrt{x}\sin(1 + x^{3/2})\,dx = \int \sin u\left(\tfrac{2}{3}\,du\right) = \tfrac{2}{3}\cdot(-\cos u) + C = -\tfrac{2}{3}\cos(1 + x^{3/2}) + C.$$

21. Let $u = \sin\theta$. Then $du = \cos\theta\,d\theta$, so $\int \cos\theta\,\sin^6\theta\,d\theta = \int u^6\,du = \frac{1}{7}u^7 + C = \frac{1}{7}\sin^7\theta + C.$

22. Let $u = 1 + \tan\theta$. Then $du = \sec^2\theta\,d\theta$, so

$$\int (1+\tan\theta)^5 \sec^2\theta\,d\theta = \int u^5\,du = \tfrac{1}{6}u^6 + C = \tfrac{1}{6}(1+\tan\theta)^6 + C.$$

23. Let $u = 1 + z^3$. Then $du = 3z^2\,dz$ and $z^2\,dz = \frac{1}{3}\,du$, so

$$\int \frac{z^2}{\sqrt[3]{1+z^3}}\,dz = \int u^{-1/3}\left(\tfrac{1}{3}\,du\right) = \tfrac{1}{3}\cdot\tfrac{3}{2}u^{2/3} + C = \tfrac{1}{2}(1+z^3)^{2/3} + C.$$

24. Let $u = ax^2 + 2bx + c$. Then $du = 2(ax+b)\,dx$ and $(ax+b)\,dx = \frac{1}{2}\,du$, so

$$\int \frac{(ax+b)\,dx}{\sqrt{ax^2+2bx+c}} = \int \frac{\frac{1}{2}\,du}{\sqrt{u}} = \tfrac{1}{2}\int u^{-1/2}\,du = u^{1/2} + C = \sqrt{ax^2+2bx+c} + C.$$

25. Let $u = \cot x$. Then $du = -\csc^2 x\,dx$ and $\csc^2 x\,dx = -du$, so

$$\int \sqrt{\cot x}\,\csc^2 x\,dx = \int \sqrt{u}\,(-du) = -\frac{u^{3/2}}{3/2} + C = -\tfrac{2}{3}(\cot x)^{3/2} + C.$$

26. Let $u = \dfrac{\pi}{x}$. Then $du = -\dfrac{\pi}{x^2}\,dx$ and $\dfrac{1}{x^2}\,dx = -\dfrac{1}{\pi}\,du$, so

$$\int \frac{\cos(\pi/x)}{x^2}\,dx = \int \cos u\left(-\frac{1}{\pi}\,du\right) = -\frac{1}{\pi}\sin u + C = -\frac{1}{\pi}\sin\frac{\pi}{x} + C.$$

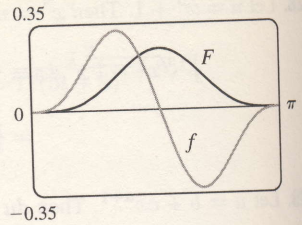
27. Let $u = \sec x$. Then $du = \sec x\,\tan x\,dx$, so

$$\int \sec^3 x\,\tan x\,dx = \int \sec^2 x\,(\sec x\,\tan x)\,dx = \int u^2\,du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}\sec^3 x + C.$$

35. $f(x) = \sin^3 x \cos x$. $u = \sin x \Rightarrow du = \cos x\, dx$, so

$$\int \sin^3 x \cos x\, dx = \int u^3\, du = \tfrac{1}{4}u^4 + C = \tfrac{1}{4}\sin^4 x + C$$

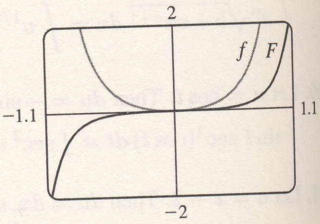
Note that at $x = \frac{\pi}{2}$, f changes from positive to negative and F has a local maximum. Also, both f and F are periodic with period π, so at $x = 0$ and at $x = \pi$, f changes from negative to positive and F has local minima.



36. $f(\theta) = \tan^2\theta \sec^2\theta$. $u = \tan\theta \Rightarrow du = \sec^2\theta\, d\theta$, so

$$\int \tan^2\theta \sec^2\theta\, d\theta = \int u^2\, du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}\tan^3\theta + C$$

Note that f is positive and F is increasing. At $x = 0$, $f = 0$ and F has a horizontal tangent.



37. Let $u = x - 1$, so $du = dx$. When $x = 0$, $u = -1$; when $x = 2$, $u = 1$. Thus, $\int_0^2 (x-1)^{25}\, dx = \int_{-1}^1 u^{25}\, du = 0$ by Theorem 6(b), since $f(u) = u^{25}$ is an odd function.

38. Let $u = 4 + 3x$, so $du = 3\, dx$. When $x = 0$, $u = 4$; when $x = 7$, $u = 25$. Thus,

$$\int_0^7 \sqrt{4+3x}\, dx = \int_4^{25} \sqrt{u}\left(\tfrac{1}{3}\, du\right) = \frac{1}{3}\left[\frac{u^{3/2}}{3/2}\right]_4^{25} = \frac{2}{9}(25^{3/2} - 4^{3/2}) = \frac{2}{9}(125 - 8) = \frac{234}{9} = 26$$

39. Let $u = 1 + 2x^3$, so $du = 6x^2\, dx$. When $x = 0$, $u = 1$; when $x = 1$, $u = 3$. Thus,

$$\int_0^1 x^2(1+2x^3)^5\, dx = \int_1^3 u^5\left(\tfrac{1}{6}\, du\right) = \tfrac{1}{6}\left[\tfrac{1}{6}u^6\right]_1^3 = \tfrac{1}{36}(3^6 - 1^6) = \tfrac{1}{36}(729 - 1) = \tfrac{728}{36} = \tfrac{182}{9}$$

40. Let $u = x^2$, so $du = 2x\, dx$. When $x = 0$, $u = 0$; when $x = \sqrt{\pi}$, $u = \pi$. Thus,

$$\int_0^{\sqrt{\pi}} x\cos(x^2)\, dx = \int_0^\pi \cos u\left(\tfrac{1}{2}\, du\right) = \tfrac{1}{2}[\sin u]_0^\pi = \tfrac{1}{2}(\sin\pi - \sin 0) = \tfrac{1}{2}(0 - 0) = 0.$$

41. Let $u = t/4$, so $du = \frac{1}{4}\, dt$. When $t = 0$, $u = 0$; when $t = \pi$, $u = \pi/4$. Thus,

$$\int_0^\pi \sec^2(t/4)\, dt = \int_0^{\pi/4} \sec^2 u\,(4\, du) = 4\big[\tan u\big]_0^{\pi/4} = 4(\tan\tfrac{\pi}{4} - \tan 0) = 4(1 - 0) = 4.$$

42. Let $u = \pi t$, so $du = \pi\, dt$. When $t = \frac{1}{6}$, $u = \frac{\pi}{6}$; when $t = \frac{1}{2}$, $u = \frac{\pi}{2}$. Thus,

$$\int_{1/6}^{1/2} \csc\pi t \cot\pi t\, dt = \int_{\pi/6}^{\pi/2} \csc u \cot u\left(\tfrac{1}{\pi}\, du\right) = \tfrac{1}{\pi}\big[-\csc u\big]_{\pi/6}^{\pi/2} = -\tfrac{1}{\pi}(1 - 2) = \tfrac{1}{\pi}.$$

43. $\int_{-\pi/6}^{\pi/6} \tan^3\theta\, d\theta = 0$ by Theorem 6(b), since $f(\theta) = \tan^3\theta$ is an odd function.

44. $\displaystyle\int_0^2 \frac{dx}{(2x-3)^2}$ does not exist since $f(x) = \dfrac{1}{(2x-3)^2}$ has an infinite discontinuity at $x = \frac{3}{2}$.

45. Let $u = \cos\theta$, so $du = -\sin\theta\, d\theta$. When $\theta = 0$, $u = 1$; when $\theta = \frac{\pi}{3}$, $u = \frac{1}{2}$. Thus,

$$\int_0^{\pi/3} \frac{\sin\theta}{\cos^2\theta}\, d\theta = \int_1^{1/2} \frac{-du}{u^2} = \int_{1/2}^1 u^{-2}\, du = \left[-\frac{1}{u}\right]_{1/2}^1 = -1 - (-2) = 1.$$

46. $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{x^2\sin x}{1+x^6}\, dx = 0$ by Theorem 6(b), since $f(x) = \dfrac{x^2\sin x}{1+x^6}$ is an odd function.

47. Let $u = 1 + 2x$, so $du = 2\, dx$. When $x = 0$, $u = 1$; when $x = 13$, $u = 27$. Thus,

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_1^{27} u^{-2/3}\left(\tfrac{1}{2}\, du\right) = \left[\tfrac{1}{2}\cdot 3u^{1/3}\right]_1^{27} = \tfrac{3}{2}(3 - 1) = 3.$$

48. Let $u = \sin x$, so $du = \cos x\,dx$. When $x = 0$, $u = 0$; when $x = \frac{\pi}{2}$, $u = 1$. Thus,

$\int_0^{\pi/2} \cos x \sin(\sin x)\,dx = \int_0^1 \sin u\,du = \left[-\cos u\right]_0^1 = -(\cos 1 - 1) = 1 - \cos 1.$

49. Let $u = x - 1$, so $u + 1 = x$ and $du = dx$. When $x = 1$, $u = 0$; when $x = 2$, $u = 1$. Thus,

$\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 (u^{3/2} + u^{1/2})\,du = \left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_0^1 = \frac{2}{5} + \frac{2}{3} = \frac{16}{15}.$

50. Let $u = 1 + 2x$, so $x = \frac{1}{2}(u-1)$ and $du = 2\,dx$. When $x = 0$, $u = 1$; when $x = 4$, $u = 9$. Thus,

$$\int_0^4 \frac{x\,dx}{\sqrt{1+2x}} = \int_1^9 \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\frac{du}{2} = \frac{1}{4}\int_1^9 (u^{1/2} - u^{-1/2})\,du = \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9$$

$$= \frac{1}{4}\cdot\frac{2}{3}\left[u^{3/2} - 3u^{1/2}\right]_1^9 = \frac{1}{6}[(27-9) - (1-3)] = \frac{20}{6} = \frac{10}{3}$$

51. $\displaystyle\int_0^4 \frac{dx}{(x-2)^3}$ does not exist since $f(x) = \dfrac{1}{(x-2)^3}$ has an infinite discontinuity at $x = 2$.

52. Assume $a > 0$. Let $u = a^2 - x^2$, so $du = -2x\,dx$. When $x = 0$, $u = a^2$; when $x = a$, $u = 0$. Thus,

$\int_0^a x\sqrt{a^2 - x^2}\,dx = \int_{a^2}^0 u^{1/2}\left(-\frac{1}{2}\,du\right) = \frac{1}{2}\int_0^{a^2} u^{1/2}\,du = \frac{1}{2}\cdot\left[\frac{2}{3}u^{3/2}\right]_0^{a^2} = \frac{1}{3}a^3.$

53. Let $u = x^2 + a^2$, so $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. When $x = 0$, $u = a^2$; when $x = a$, $u = 2a^2$. Thus,

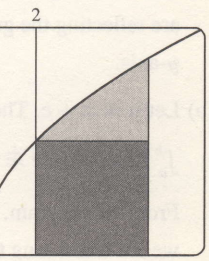
$$\int_0^a x\sqrt{x^2 + a^2}\,dx = \int_{a^2}^{2a^2} u^{1/2}\left(\tfrac{1}{2}\,du\right) = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_{a^2}^{2a^2} = \left[\frac{1}{3}u^{3/2}\right]_{a^2}^{2a^2}$$

$$= \frac{1}{3}\left[(2a^2)^{3/2} - (a^2)^{3/2}\right] = \frac{1}{3}\left(2\sqrt{2} - 1\right)a^3$$

54. $\int_{-a}^a x\sqrt{x^2 + a^2}\,dx = 0$ by Theorem 6(b), since $f(x) = x\sqrt{x^2 + a^2}$ is an odd function.

55. From the graph, it appears that the area under the curve is about $1 + \left(\text{a little more than } \frac{1}{2}\cdot 1\cdot 0.7\right)$, or about 1.4. The exact area is given by $A = \int_0^1 \sqrt{2x+1}\,dx$. Let $u = 2x + 1$, so $du = 2\,dx$. The limits change to $2\cdot 0 + 1 = 1$ and $2\cdot 1 + 1 = 3$, and

$A = \int_1^3 \sqrt{u}\left(\frac{1}{2}\,du\right) = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^3 = \frac{1}{3}(3\sqrt{3} - 1) = \sqrt{3} - \frac{1}{3} \approx 1.399.$

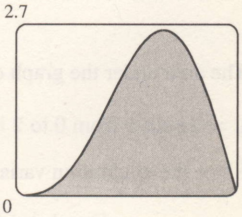


56. From the graph, it appears that the area under the curve is almost $\frac{1}{2}\cdot\pi\cdot 2.6$, or about 4. The exact area is given by

$A = \int_0^\pi (2\sin x - \sin 2x)dx = -2\left[\cos x\right]_0^\pi - \int_0^\pi \sin 2x\,dx$

$= -2(-1-1) - 0 = 4$

Note: $\int_0^\pi \sin 2x\,dx = 0$ since it is clear from the graph of $y = \sin 2x$ that $\int_{\pi/2}^\pi \sin 2x\,dx = -\int_0^{\pi/2} \sin 2x\,dx$.



66. Let $u = \pi - x$. Then $du = -dx$. When $x = \pi$, $u = 0$ and when $x = 0$, $u = \pi$. So

$$\int_0^\pi xf(\sin x)\,dx = -\int_\pi^0 (\pi - u)f(\sin(\pi - u))\,du = \int_0^\pi (\pi - u)f(\sin u)\,du$$

$$= \pi\int_0^\pi f(\sin u)\,du - \int_0^\pi uf(\sin u)\,du = \pi\int_0^\pi f(\sin x)\,dx - \int_0^\pi xf(\sin x)\,dx$$

$$\Rightarrow\quad 2\int_0^\pi xf(\sin x)\,dx = \pi\int_0^\pi f(\sin x)\,dx \quad\Rightarrow\quad \int_0^\pi xf(\sin x)\,dx = \frac{\pi}{2}\int_0^\pi f(\sin x)\,dx.$$

67. Let $u = 5 - 3x$. Then $du = -3\,dx$ and $dx = -\frac{1}{3}\,du$, so

$$\int \frac{dx}{5 - 3x} = \int \frac{1}{u}\left(-\tfrac{1}{3}\,du\right) = -\tfrac{1}{3}\ln|u| + C = -\tfrac{1}{3}\ln|5 - 3x| + C.$$

68. Let $u = x^2 + 1$. Then $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$, so

$$\int \frac{x}{x^2+1}\,dx = \int \frac{\frac{1}{2}\,du}{u} = \tfrac{1}{2}\ln|u| + C = \tfrac{1}{2}\ln|x^2+1| + C = \tfrac{1}{2}\ln(x^2+1) + C \quad [\text{since } x^2 + 1 > 0]$$

or $\ln(x^2+1)^{1/2} + C = \ln\sqrt{x^2+1} + C$.

69. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$, so $\displaystyle\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}(\ln x)^3 + C$.

70. Let $u = \tan^{-1} x$. Then $du = \dfrac{dx}{1+x^2}$, so $\displaystyle\int \frac{\tan^{-1} x}{1+x^2}\,dx = \int u\,du = \frac{u^2}{2} + C = \frac{(\tan^{-1} x)^2}{2} + C$.

71. Let $u = 1 + e^x$. Then $du = e^x\,dx$, so $\int e^x\sqrt{1+e^x}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(1+e^x)^{3/2} + C$.

Or: Let $u = \sqrt{1+e^x}$. Then $u^2 = 1 + e^x$ and $2u\,du = e^x\,dx$, so

$\int e^x\sqrt{1+e^x}\,dx = \int u\cdot 2u\,du = \frac{2}{3}u^3 + C = \frac{2}{3}(1+e^x)^{3/2} + C$.

72. Let $u = \cos t$. Then $du = -\sin t\,dt$ and $\sin t\,dt = -du$, so

$\int e^{\cos t}\sin t\,dt = \int e^u\,(-du) = -e^u + C = -e^{\cos t} + C$.

73. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$, so $\displaystyle\int \frac{dx}{x\ln x} = \int \frac{du}{u} = \ln|u| + C = \ln|\ln x| + C$.

74. Let $u = e^x + 1$. Then $du = e^x\,dx$, so $\displaystyle\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x+1) + C$.

75. $\displaystyle\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$. Let $u = \sin x$. Then $du = \cos x\,dx$, so

$\int \cot x\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\sin x| + C$.

76. Let $u = \cos x$. Then $du = -\sin x\,dx$ and $\sin x\,dx = -du$, so

$$\int \frac{\sin x}{1+\cos^2 x}\,dx = \int \frac{-du}{1+u^2} = -\tan^{-1} u + C = -\tan^{-1}(\cos x) + C.$$

77. Let $u = 1 + x^2$. Then $du = 2x\,dx$, so

$$\int \frac{1+x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx = \tan^{-1} x + \int \frac{\frac{1}{2}\,du}{u} = \tan^{-1} x + \tfrac{1}{2}\ln|u| + C$$

$$= \tan^{-1} x + \tfrac{1}{2}\ln\left|1+x^2\right| + C = \tan^{-1} x + \tfrac{1}{2}\ln\left(1+x^2\right) + C \quad [\text{since } 1 + x^2 > 0].$$