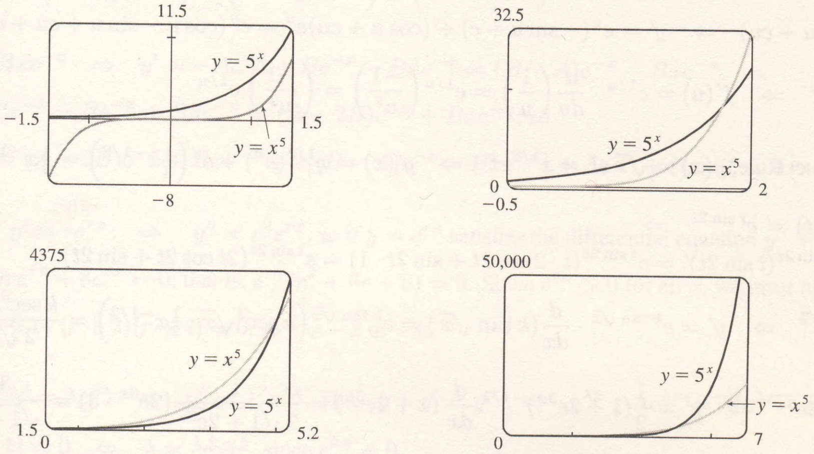
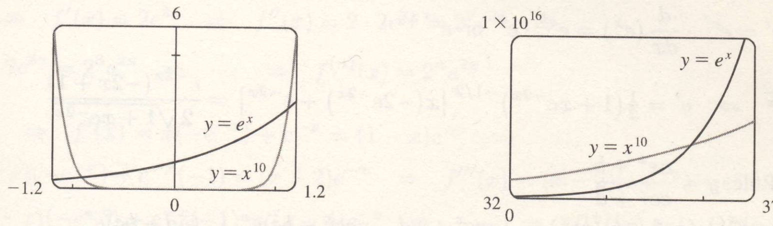
18. Given the y-intercept $(0, 2)$, we have $y = Ca^x = 2a^x$. Using the point $\left(2, \frac{2}{9}\right)$ gives us $\frac{2}{9} = 2a^2 \Rightarrow \frac{1}{9} = a^2$ $\Rightarrow a = \frac{1}{3}$ [since $a > 0$]. The function is $f(x) = 2\left(\frac{1}{3}\right)^x$ or $f(x) = 2(3)^{-x}$.

19. 2 ft = 24 in, $f(24) = 24^2$ in $= 576$ in $= 48$ ft. $g(24) = 2^{24}$ in $= 2^{24}/(12 \cdot 5280)$ mi ≈ 265 mi

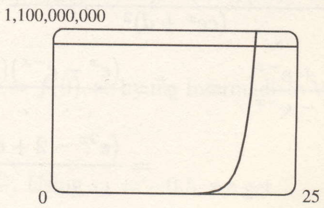
20. We see from the graphs that for x less than about 1.8, $g(x) = 5^x > f(x) = x^5$, and then near the point $(1.8, 17.1)$ the curves intersect. Then $f(x) > g(x)$ from $x \approx 1.8$ until $x = 5$. At $(5, 3125)$ there is another point of intersection, and for $x > 5$ we see that $g(x) > f(x)$. In fact, g increases much more rapidly than f beyond that point.



21. The graph of g finally surpasses that of f at $x \approx 35.8$.



22. We graph $y = e^x$ and $y = 1{,}000{,}000{,}000$ and determine where $e^x = 1 \times 10^9$. This seems to be true at $x \approx 20.723$, so $e^x > 1 \times 10^9$ for $x > 20.723$.



23. $\lim\limits_{x \to \infty} (1.001)^x = \infty$ by (3), since $1.001 > 1$.

24. Let $t = -x^2$. As $x \to \infty$, $t \to -\infty$. So $\lim\limits_{x \to \infty} e^{-x^2} = \lim\limits_{t \to -\infty} e^t = 0$ by (11).

25. Divide numerator and denominator by e^{3x}: $\lim\limits_{x \to \infty} \dfrac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \lim\limits_{x \to \infty} \dfrac{1 - e^{-6x}}{1 + e^{-6x}} = \dfrac{1 - 0}{1 + 0} = 1$

26. If we let $t = \tan x$, then as $x \to (\pi/2)^+$, $t \to -\infty$. Thus, $\lim\limits_{x \to (\pi/2)^+} e^{\tan x} = \lim\limits_{t \to -\infty} e^t = 0$.

27. Let $t = 3/(2 - x)$. As $x \to 2^+$, $t \to -\infty$. So $\lim\limits_{x \to 2^+} e^{3/(2-x)} = \lim\limits_{t \to -\infty} e^t = 0$ by (11).

28. Let $t = 3/(2-x)$. As $x \to 2^-$, $t \to \infty$. So $\lim\limits_{x\to 2^-} e^{3/(2-x)} = \lim\limits_{t\to\infty} e^t = \infty$ by (11).

29. By the Product Rule, $f(x) = x^2 e^x \Rightarrow f'(x) = x^2 \dfrac{d}{dx}(e^x) + e^x \dfrac{d}{dx}(x^2) = x^2 e^x + e^x(2x) = xe^x(x+2)$.

30. By the Quotient Rule, $y = \dfrac{e^x}{1+x} \Rightarrow y' = \dfrac{(1+x)\,e^x - e^x(1)}{(1+x)^2} = \dfrac{e^x + xe^x - e^x}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}$.

31. By (10), $y = e^{ax^3} \Rightarrow y' = e^{ax^3}\dfrac{d}{dx}(ax^3) = 3ax^2 e^{ax^3}$.

32. $y = e^u(\cos u + cu) \Rightarrow y' = e^u(-\sin u + c) + (\cos u + cu)e^u = e^u(\cos u - \sin u + cu + c)$

33. $f(u) = e^{1/u} \Rightarrow f'(u) = e^{1/u}\cdot \dfrac{d}{du}\left(\dfrac{1}{u}\right) = e^{1/u}\left(\dfrac{-1}{u^2}\right) = \left(\dfrac{-1}{u^2}\right)e^{1/u}$

34. By the Product Rule, $g(x) = \sqrt{x}\,e^x = x^{1/2}e^x \Rightarrow g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x+1)$.

35. By (10), $F(t) = e^{t\sin 2t} \Rightarrow$

$F'(t) = e^{t\sin 2t}(t\sin 2t)' = e^{t\sin 2t}(t\cdot 2\cos 2t + \sin 2t\cdot 1) = e^{t\sin 2t}(2t\cos 2t + \sin 2t)$

36. $y = e^{k\tan\sqrt{x}} \Rightarrow y' = e^{k\tan\sqrt{x}}\cdot\dfrac{d}{dx}(k\tan\sqrt{x}) = e^{k\tan\sqrt{x}}\left(k\sec^2\sqrt{x}\cdot\frac{1}{2}x^{-1/2}\right) = \dfrac{k\sec^2\sqrt{x}}{2\sqrt{x}}\,e^{k\tan\sqrt{x}}$

37. $y = \sqrt{1+2e^{3x}} \Rightarrow y' = \dfrac{1}{2}(1+2e^{3x})^{-1/2}\dfrac{d}{dx}(1+2e^{3x}) = \dfrac{1}{2\sqrt{1+2e^{3x}}}(2e^{3x}\cdot 3) = \dfrac{3e^{3x}}{\sqrt{1+2e^{3x}}}$

38. $y = \cos(e^{\pi x}) \Rightarrow y' = -\sin(e^{\pi x})\cdot e^{\pi x}\cdot\pi = -\pi e^{\pi x}\sin(e^{\pi x})$

39. $y = e^{e^x} \Rightarrow y' = e^{e^x}\cdot\dfrac{d}{dx}(e^x) = e^{e^x}\cdot e^x$ or e^{e^x+x}

40. $y = \sqrt{1+xe^{-2x}} \Rightarrow y' = \frac{1}{2}\left(1+xe^{-2x}\right)^{-1/2}\left[x(-2e^{-2x}) + e^{-2x}\right] = \dfrac{e^{-2x}(-2x+1)}{2\sqrt{1+xe^{-2x}}}$

41. By the Quotient Rule, $y = \dfrac{ae^x+b}{ce^x+d} \Rightarrow$

$y' = \dfrac{(ce^x+d)(ae^x) - (ae^x+b)(ce^x)}{(ce^x+d)^2} = \dfrac{(ace^x + ad - ace^x - bc)e^x}{(ce^x+d)^2} = \dfrac{(ad-bc)e^x}{(ce^x+d)^2}$.

42. $y = \dfrac{e^x+e^{-x}}{e^x-e^{-x}} \Rightarrow y' = \dfrac{(e^x-e^{-x})(e^x-e^{-x}) - (e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}$

$= \dfrac{(e^{2x}-2+e^{-2x}) - (e^{2x}+2+e^{-2x})}{(e^x-e^{-x})^2} = -\dfrac{4}{(e^x-e^{-x})^2}$

43. $y = e^{2x}\cos\pi x \Rightarrow y' = e^{2x}(-\pi\sin\pi x) + (\cos\pi x)(2e^{2x}) = e^{2x}(2\cos\pi x - \pi\sin\pi x)$.
At $(0,1)$, $y' = 1(2-0) = 2$, so an equation of the tangent line is $y-1 = 2(x-0)$, or $y = 2x+1$.

44. $y = \dfrac{e^x}{x} \Rightarrow y' = \dfrac{x\cdot e^x - e^x\cdot 1}{x^2} = \dfrac{e^x(x-1)}{x^2}$. At $(1,e)$, $y' = 0$, and an equation of the tangent line is $y - e = 0(x-1)$, or $y = e$.

45. $\dfrac{d}{dx}\left(e^{x^2y}\right) = \dfrac{d}{dx}(x+y) \Rightarrow e^{x^2y}(x^2y' + y\cdot 2x) = 1 + y' \Rightarrow x^2e^{x^2y}y' + 2xye^{x^2y} = 1 + y' \Rightarrow$

$x^2e^{x^2y}y' - y' = 1 - 2xye^{x^2y} \Rightarrow y'(x^2e^{x^2y} - 1) = 1 - 2xye^{x^2y} \Rightarrow y' = \dfrac{1-2xye^{x^2y}}{x^2e^{x^2y}-1}$