11. $F(x)=\int_x^2 \cos(t^2)\,dt = -\int_2^x \cos(t^2)\,dt \quad\Rightarrow\quad F'(x) = -\cos(x^2)$

12. $f(\theta)=\tan\theta$ and $F(x)=\int_x^{10}\tan\theta\,d\theta = -\int_{10}^x \tan\theta\,d\theta$, so by FTC1, $F'(x) = -f(x) = -\tan x$.

13. Let $u=\dfrac{1}{x}$. Then $\dfrac{du}{dx}=-\dfrac{1}{x^2}$. Also, $\dfrac{dh}{dx}=\dfrac{dh}{du}\dfrac{du}{dx}$, so

$$h'(x)=\frac{d}{dx}\int_2^{1/x}\sin^4 t\,dt=\frac{d}{du}\int_2^u \sin^4 t\,dt\cdot\frac{du}{dx}=\sin^4 u\,\frac{du}{dx}=\frac{-\sin^4(1/x)}{x^2}.$$

14. Let $u=x^2$. Then $\dfrac{du}{dx}=2x$. Also, $\dfrac{dh}{dx}=\dfrac{dh}{du}\dfrac{du}{dx}$, so

$$h'(x)=\frac{d}{dx}\int_0^{x^2}\sqrt{1+r^3}\,dr=\frac{d}{du}\int_0^u\sqrt{1+r^3}\,dr\cdot\frac{du}{dx}=\sqrt{1+u^3}\,(2x)=2x\sqrt{1+(x^2)^3}=2x\sqrt{1+x^6}.$$

15. Let $u=\sqrt{x}$. Then $\dfrac{du}{dx}=\dfrac{1}{2\sqrt{x}}$. Also, $\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$, so

$$y'=\frac{d}{dx}\int_3^{\sqrt{x}}\frac{\cos t}{t}\,dt=\frac{d}{du}\int_3^u\frac{\cos t}{t}\,dt\cdot\frac{du}{dx}=\frac{\cos u}{u}\cdot\frac{1}{2\sqrt{x}}=\frac{\cos\sqrt{x}}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}=\frac{\cos\sqrt{x}}{2x}.$$

16. Let $u=\cos x$. Then $\dfrac{du}{dx}=-\sin x$. Also, $\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$, so

$$y'=\frac{d}{dx}\int_1^{\cos x}(t+\sin t)\,dt=\frac{d}{du}\int_1^u(t+\sin t)\,dt\cdot\frac{du}{dx}$$
$$=(u+\sin u)\cdot(-\sin x)=-\sin x\,[\cos x+\sin(\cos x)]$$

17. Let $w=1-3x$. Then $\dfrac{dw}{dx}=-3$. Also, $\dfrac{dy}{dx}=\dfrac{dy}{dw}\dfrac{dw}{dx}$, so

$$y'=\frac{d}{dx}\int_{1-3x}^1\frac{u^3}{1+u^2}\,du=\frac{d}{dw}\int_w^1\frac{u^3}{1+u^2}\,du\cdot\frac{dw}{dx}$$
$$=-\frac{d}{dw}\int_1^w\frac{u^3}{1+u^2}\,du\cdot\frac{dw}{dx}=-\frac{w^3}{1+w^2}(-3)=\frac{3(1-3x)^3}{1+(1-3x)^2}$$

18. Let $u=\dfrac{1}{x^2}$. Then $\dfrac{du}{dx}=-\dfrac{2}{x^3}$. Also, $\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$, so

$$y'=\frac{d}{dx}\int_{1/x^2}^0\sin^3 t\,dt=\frac{d}{du}\int_u^0\sin^3 t\,dt\cdot\frac{du}{dx}=-\frac{d}{du}\int_0^u\sin^3 t\,dt\cdot\frac{du}{dx}=-\sin^3 u\left(-\frac{2}{x^3}\right)=\frac{2\sin^3(1/x^2)}{x^3}.$$

19. $\displaystyle\int_{-1}^3 x^5\,dx=\left[\frac{x^6}{6}\right]_{-1}^3=\frac{3^6}{6}-\frac{(-1)^6}{6}=\frac{729-1}{6}=\frac{364}{3}$

20. $\int_{-2}^5 6\,dx=[6x]_{-2}^5=6[5-(-2)]=6(7)=42$

21. $\int_2^8(4x+3)\,dx=\left[\frac{4}{2}x^2+3x\right]_2^8=(2\cdot 8^2+3\cdot 8)-(2\cdot 2^2+3\cdot 2)=152-14=138$

22. $\int_0^4(1+3y-y^2)\,dy=\left[y+\frac{3}{2}y^2-\frac{1}{3}y^3\right]_0^4=\left(4+\frac{3}{2}\cdot 16-\frac{1}{3}\cdot 64\right)-(0)=\frac{20}{3}$

23. $\int_0^1 x^{4/5}\,dx=\left[\frac{5}{9}x^{9/5}\right]_0^1=\frac{5}{9}-0=\frac{5}{9}$

24. $\int_1^8\sqrt[3]{x}\,dx=\int_1^8 x^{1/3}\,dx=\left[\frac{3}{4}x^{4/3}\right]_1^8=\frac{3}{4}(8^{4/3}-1^{4/3})=\frac{3}{4}(2^4-1)=\frac{3}{4}(16-1)=\frac{3}{4}(15)=\frac{45}{4}$

25. $\displaystyle\int_1^2\frac{3}{t^4}\,dt=3\int_1^2 t^{-4}\,dt=3\left[\frac{t^{-3}}{-3}\right]_1^2=\frac{3}{-3}\left[\frac{1}{t^3}\right]_1^2=-1\left(\frac{1}{8}-1\right)=\frac{7}{8}$

26. $\int_{-2}^3 x^{-5}\,dx$ …
discontinuo…

27. $\displaystyle\int_{-5}^5\frac{2}{x^3}\,dx$ …
discontinu…

28. $\int_\pi^{2\pi}\cos\theta$ …

29. $\int_0^2 x(2+x$ …

30. $\displaystyle\int_1^4\frac{1}{\sqrt{x}}\,d$ …

31. $\int_0^{\pi/4}\sec^2$ …

32. $\int_0^1(3+x$ …

33. $\int_\pi^{2\pi}\csc^2$ …
that is, f i…

34. $\int_0^{\pi/6}\csc$ …
that is, f i…

35. $\int_0^2 f(x)$ …

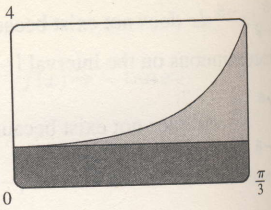
36. $\int_{-\pi}^\pi f(x)$ …

37. From the …
$\int_0^{27}x^{1/3}$ …
area of th…

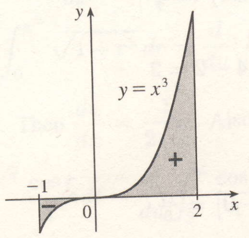
38. From the …
$\displaystyle\int_1^6 x^{-4}$ …

39. It appea…
viewing …
$\int_0^\pi \sin$ …

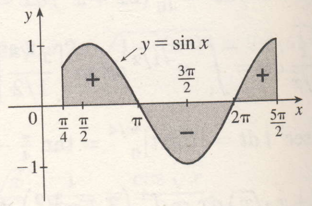
40. Splitting up the region as shown, we estimate that the area under the graph is $\frac{\pi}{3} + \frac{1}{4}\left(3 \cdot \frac{\pi}{3}\right) \approx 1.8$. The actual area is $\int_0^{\pi/3} \sec^2 x\,dx = [\tan x]_0^{\pi/3} = \sqrt{3} - 0 = \sqrt{3} \approx 1.73$.



41. $\int_{-1}^{2} x^3\,dx = \left[\frac{1}{4}x^4\right]_{-1}^{2} = 4 - \frac{1}{4} = \frac{15}{4} = 3.75$



42. $\int_{\pi/4}^{5\pi/2} \sin x\,dx = [-\cos x]_{\pi/4}^{5\pi/2} = 0 + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$



43. $g(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\,du = \int_{2x}^{0} \frac{u^2 - 1}{u^2 + 1}\,du + \int_{0}^{3x} \frac{u^2 - 1}{u^2 + 1}\,du = -\int_{0}^{2x} \frac{u^2 - 1}{u^2 + 1}\,du + \int_{0}^{3x} \frac{u^2 - 1}{u^2 + 1}\,du \Rightarrow$

$g'(x) = -\frac{(2x)^2 - 1}{(2x)^2 + 1} \cdot \frac{d}{dx}(2x) + \frac{(3x)^2 - 1}{(3x)^2 + 1} \cdot \frac{d}{dx}(3x) = -2 \cdot \frac{4x^2 - 1}{4x^2 + 1} + 3 \cdot \frac{9x^2 - 1}{9x^2 + 1}$

44. $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2 + t^4}}\,dt = \int_{\tan x}^{1} \frac{dt}{\sqrt{2 + t^4}} + \int_{1}^{x^2} \frac{dt}{\sqrt{2 + t^4}} = -\int_{1}^{\tan x} \frac{dt}{\sqrt{2 + t^4}} + \int_{1}^{x^2} \frac{dt}{\sqrt{2 + t^4}} \Rightarrow$

$g'(x) = \frac{-1}{\sqrt{2 + \tan^4 x}} \frac{d}{dx}(\tan x) + \frac{1}{\sqrt{2 + x^8}} \frac{d}{dx}(x^2) = -\frac{\sec^2 x}{\sqrt{2 + \tan^4 x}} + \frac{2x}{\sqrt{2 + x^8}}$

45. $y = \int_{\sqrt{x}}^{x^3} \sqrt{t}\sin t\,dt = \int_{\sqrt{x}}^{1} \sqrt{t}\sin t\,dt + \int_{1}^{x^3} \sqrt{t}\sin t\,dt = -\int_{1}^{\sqrt{x}} \sqrt{t}\sin t\,dt + \int_{1}^{x^3} \sqrt{t}\sin t\,dt \Rightarrow$

$y' = -\sqrt[4]{x}\,(\sin\sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x}) + x^{3/2}\sin(x^3) \cdot \frac{d}{dx}(x^3) = -\frac{\sqrt[4]{x}\sin\sqrt{x}}{2\sqrt{x}} + x^{3/2}\sin(x^3)\,(3x^2)$

$= 3x^{7/2}\sin(x^3) - \frac{\sin\sqrt{x}}{2\sqrt[4]{x}}$

46. $y = \int_{\cos x}^{5x} \cos(u^2)\,du = \int_0^{5x} \cos(u^2)\,du - \int_0^{\cos x} \cos(u^2)\,du \Rightarrow$

$y' = \cos(25x^2) \cdot \frac{d}{dx}(5x) - \cos(\cos^2 x) \cdot \frac{d}{dx}(\cos x) = \cos(25x^2) \cdot 5 - \cos(\cos^2 x) \cdot (-\sin x)$

$= 5\cos(25x^2) + \sin x \cos(\cos^2 x)$

47. $F(x) = \int_1^x f(t)\,dt \quad \Rightarrow \quad F'(x) = f(x) = \int_1^{x^2} \frac{\sqrt{1 + u^4}}{u}\,du \quad \left[\text{since } f(t) = \int_1^{t^2} \frac{\sqrt{1 + u^4}}{u}\,du\right] \quad \Rightarrow$

$F''(x) = f'(x) = \frac{\sqrt{1 + (x^2)^4}}{x^2} \cdot \frac{d}{dx}(x^2) = \frac{\sqrt{1 + x^8}}{x^2} \cdot 2x = \frac{2\sqrt{1 + x^8}}{x}$. So $F''(2) = \sqrt{1 + 2^8} = \sqrt{257}$.

48. For the curve to be concave upward, we must have $y'' > 0$. $y = \int_0^x \frac{1}{1 + t + t^2}\,dt \quad \Rightarrow \quad y' = \frac{1}{1 + x + x^2} \quad \Rightarrow$

$y'' = \frac{-(1 + 2x)}{(1 + x + x^2)^2}$. For this expression to be positive, we must have $(1 + 2x) < 0$, since $(1 + x + x^2)^2 > 0$ for all x. $(1 + 2x) < 0 \quad \Leftrightarrow \quad x < -\frac{1}{2}$. Thus, the curve is concave upward on $\left(-\infty, -\frac{1}{2}\right)$.