6. $\lim\limits_{x\to -2} \dfrac{x+2}{x^2+3x+2} = \lim\limits_{x\to -2} \dfrac{x+2}{(x+1)(x+2)} = \lim\limits_{x\to -2} \dfrac{1}{x+1} = -1$

7. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 1} \dfrac{x^9-1}{x^5-1} \overset{\text{H}}{=} \lim\limits_{x\to 1} \dfrac{9x^8}{5x^4} = \dfrac{9}{5} \lim\limits_{x\to 1} x^4 = \dfrac{9}{5}(1) = \dfrac{9}{5}$

8. $\lim\limits_{x\to 1} \dfrac{x^a-1}{x^b-1} \overset{\text{H}}{=} \lim\limits_{x\to 1} \dfrac{ax^{a-1}}{bx^{b-1}} = \dfrac{a}{b}$

9. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to(\pi/2)^+} \dfrac{\cos x}{1-\sin x} \overset{\text{H}}{=} \lim\limits_{x\to(\pi/2)^+} \dfrac{-\sin x}{-\cos x} = \lim\limits_{x\to(\pi/2)^+} \tan x = -\infty$.

10. $\lim\limits_{x\to 0} \dfrac{x+\tan x}{\sin x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{1+\sec^2 x}{\cos x} = \dfrac{1+1^2}{1} = 2$

11. This limit has the form $\frac{0}{0}$. $\lim\limits_{t\to 0} \dfrac{e^t-1}{t^3} \overset{\text{H}}{=} \lim\limits_{t\to 0} \dfrac{e^t}{3t^2} = \infty$ since $e^t \to 1$ and $3t^2 \to 0^+$ as $t \to 0$.

12. $\lim\limits_{t\to 0} \dfrac{e^{3t}-1}{t} \overset{\text{H}}{=} \lim\limits_{t\to 0} \dfrac{3e^{3t}}{1} = 3$

13. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 0} \dfrac{\tan px}{\tan qx} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{p\sec^2 px}{q\sec^2 qx} = \dfrac{p(1)^2}{q(1)^2} = \dfrac{p}{q}$

14. $\lim\limits_{\theta\to\pi/2} \dfrac{1-\sin\theta}{\csc\theta} = \dfrac{0}{1} = 0$. L'Hospital's Rule does not apply.

15. This limit has the form $\frac{\infty}{\infty}$. $\lim\limits_{x\to\infty} \dfrac{\ln x}{x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{1/x}{1} = 0$

16. $\lim\limits_{x\to\infty} \dfrac{e^x}{x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{e^x}{1} = \lim\limits_{x\to\infty} e^x = \infty$

17. $\lim\limits_{x\to 0^+} [(\ln x)/x] = -\infty$ since $\ln x \to -\infty$ as $x \to 0^+$ and dividing by small values of x just increases the magnitude of the quotient $(\ln x)/x$. L'Hospital's Rule does not apply.

18. $\lim\limits_{x\to\infty} \dfrac{\ln\ln x}{x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1} = \lim\limits_{x\to\infty} \dfrac{1}{x\ln x} = 0$

19. This limit has the form $\frac{0}{0}$. $\lim\limits_{t\to 0} \dfrac{5^t-3^t}{t} \overset{\text{H}}{=} \lim\limits_{t\to 0} \dfrac{5^t\ln 5 - 3^t\ln 3}{1} = \ln 5 - \ln 3 = \ln\frac{5}{3}$

20. $\lim\limits_{x\to 1} \dfrac{\ln x}{\sin\pi x} \overset{\text{H}}{=} \lim\limits_{x\to 1} \dfrac{1/x}{\pi\cos\pi x} = \dfrac{1}{\pi(-1)} = -\dfrac{1}{\pi}$

21. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 0} \dfrac{e^x-1-x}{x^2} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{e^x-1}{2x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{e^x}{2} = \dfrac{1}{2}$

22. $\lim\limits_{x\to 0} \dfrac{e^x-1-x-x^2/2}{x^3} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{e^x-1-x}{3x^2} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{e^x-1}{6x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{e^x}{6} = \dfrac{1}{6}$

23. This limit has the form $\frac{\infty}{\infty}$. $\lim\limits_{x\to\infty} \dfrac{e^x}{x^3} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{e^x}{3x^2} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{e^x}{6x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{e^x}{6} = \infty$

24. $\lim\limits_{x\to 0} \dfrac{\sin x}{\sinh x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{\cos x}{\cosh x} = \dfrac{1}{1} = 1$

25. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 0} \dfrac{\sin^{-1} x}{x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{1/\sqrt{1-x^2}}{1} = \lim\limits_{x\to 0} \dfrac{1}{\sqrt{1-x^2}} = \dfrac{1}{1} = 1$

26. $\lim\limits_{x\to 0} \dfrac{\sin x - x}{x^3} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{\cos x - 1}{3x^2} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{-\sin x}{6x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{-\cos x}{6} = -\dfrac{1}{6}$

27. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{\sin x}{2x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{\cos x}{2} = \dfrac{1}{2}$

28. $\displaystyle\lim_{x\to\infty}\frac{(\ln x)^2}{x}\overset{\text{H}}{=}\lim_{x\to\infty}\frac{2(\ln x)(1/x)}{1}=2\lim_{x\to\infty}\frac{\ln x}{x}\overset{\text{H}}{=}2\lim_{x\to\infty}\frac{1/x}{1}=2(0)=0$

29. $\displaystyle\lim_{x\to 0}\frac{x+\sin x}{x+\cos x}=\frac{0+0}{0+1}=\frac{0}{1}=0.$ L'Hospital's Rule does not apply.

30. $\displaystyle\lim_{x\to 0}\frac{\cos mx-\cos nx}{x^2}\overset{\text{H}}{=}\lim_{x\to 0}\frac{-m\sin mx+n\sin nx}{2x}\overset{\text{H}}{=}\lim_{x\to 0}\frac{-m^2\cos mx+n^2\cos nx}{2}=\tfrac{1}{2}(n^2-m^2)$

31. This limit has the form $\frac{\infty}{\infty}$. $\displaystyle\lim_{x\to\infty}\frac{x}{\ln(1+2e^x)}\overset{\text{H}}{=}\lim_{x\to\infty}\frac{1}{\dfrac{1}{1+2e^x}\cdot 2e^x}=\lim_{x\to\infty}\frac{1+2e^x}{2e^x}\overset{\text{H}}{=}\lim_{x\to\infty}\frac{2e^x}{2e^x}=1$

32. $\displaystyle\lim_{x\to 0}\frac{x}{\tan^{-1}(4x)}\overset{\text{H}}{=}\lim_{x\to 0}\frac{1}{\dfrac{1}{1+(4x)^2}\cdot 4}=\lim_{x\to 0}\frac{1+16x^2}{4}=\frac{1}{4}$

33. This limit has the form $\frac{0}{0}$. $\displaystyle\lim_{x\to 1}\frac{1-x+\ln x}{1+\cos\pi x}\overset{\text{H}}{=}\lim_{x\to 1}\frac{-1+1/x}{-\pi\sin\pi x}\overset{\text{H}}{=}\lim_{x\to 1}\frac{-1/x^2}{-\pi^2\cos\pi x}=\frac{-1}{-\pi^2(-1)}=\frac{1}{\pi^2}$

34. $\displaystyle\lim_{x\to\infty}\frac{\sqrt{x^2+2}}{\sqrt{2x^2+1}}=\lim_{x\to\infty}\sqrt{\frac{x^2+2}{2x^2+1}}=\sqrt{\lim_{x\to\infty}\frac{x^2+2}{2x^2+1}}=\sqrt{\lim_{x\to\infty}\frac{1+2/x^2}{2+1/x^2}}=\sqrt{\frac{1}{2}}$

35. This limit has the form $\frac{0}{0}$. $\displaystyle\lim_{x\to 1}\frac{x^a-ax+a-1}{(x-1)^2}\overset{\text{H}}{=}\lim_{x\to 1}\frac{ax^{a-1}-a}{2(x-1)}\overset{\text{H}}{=}\lim_{x\to 1}\frac{a(a-1)x^{a-2}}{2}=\frac{a(a-1)}{2}$

36. $\displaystyle\lim_{x\to 0}\frac{1-e^{-2x}}{\sec x}=\frac{1-1}{1}=0.$ L'Hospital's Rule does not apply.

37. This limit has the form $0\cdot(-\infty)$. We need to write this product as a quotient, but keep in mind that we will have to differentiate both the numerator and the denominator. If we differentiate $\dfrac{1}{\ln x}$, we get a complicated expression that results in a more difficult limit. Instead we write the quotient as $\dfrac{\ln x}{x^{-1/2}}$.

$$\lim_{x\to 0^+}\sqrt{x}\ln x=\lim_{x\to 0^+}\frac{\ln x}{x^{-1/2}}\overset{\text{H}}{=}\lim_{x\to 0^+}\frac{1/x}{-\frac{1}{2}x^{-3/2}}\cdot\frac{-2x^{3/2}}{-2x^{3/2}}=\lim_{x\to 0^+}\left(-2\sqrt{x}\right)=0$$

38. $\displaystyle\lim_{x\to-\infty}x^2e^x=\lim_{x\to-\infty}\frac{x^2}{e^{-x}}\overset{\text{H}}{=}\lim_{x\to-\infty}\frac{2x}{-e^{-x}}\overset{\text{H}}{=}\lim_{x\to-\infty}\frac{2}{e^{-x}}=\lim_{x\to-\infty}2e^x=0$

39. This limit has the form $\infty\cdot 0$. We'll change it to the form $\frac{0}{0}$.

$$\lim_{x\to 0}\cot 2x\sin 6x=\lim_{x\to 0}\frac{\sin 6x}{\tan 2x}\overset{\text{H}}{=}\lim_{x\to 0}\frac{6\cos 6x}{2\sec^2 2x}=\frac{6(1)}{2(1)^2}=3$$

40. $\displaystyle\lim_{x\to 0^+}\sin x\ln x=\lim_{x\to 0^+}\frac{\ln x}{\csc x}\overset{\text{H}}{=}\lim_{x\to 0^+}\frac{1/x}{-\csc x\cot x}=-\lim_{x\to 0^+}\left(\frac{\sin x}{x}\cdot\tan x\right)$

$$=-\left(\lim_{x\to 0^+}\frac{\sin x}{x}\right)\left(\lim_{x\to 0^+}\tan x\right)=-1\cdot 0=0$$

41. This limit has the form $\infty\cdot 0$. $\displaystyle\lim_{x\to\infty}x^3e^{-x^2}=\lim_{x\to\infty}\frac{x^3}{e^{x^2}}\overset{\text{H}}{=}\lim_{x\to\infty}\frac{3x^2}{2xe^{x^2}}=\lim_{x\to\infty}\frac{3x}{2e^{x^2}}\overset{\text{H}}{=}\lim_{x\to\infty}\frac{3}{4xe^{x^2}}=0$

42. $\displaystyle\lim_{x\to\pi/4}(1-\tan x)\sec x=(1-1)\sqrt{2}=0.$ L'Hospital's Rule does not apply.

43. This limit has the form $0\cdot(-\infty)$.

$$\lim_{x\to 1^+}\ln x\,\tan(\pi x/2)=\lim_{x\to 1^+}\frac{\ln x}{\cot(\pi x/2)}\overset{\text{H}}{=}\lim_{x\to 1^+}\frac{1/x}{(-\pi/2)\csc^2(\pi x/2)}=\frac{1}{(-\pi/2)(1)^2}=-\frac{2}{\pi}$$

44. $\lim\limits_{x\to\infty} x\tan(1/x) = \lim\limits_{x\to\infty} \dfrac{\tan(1/x)}{1/x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{\sec^2(1/x)(-1/x^2)}{-1/x^2} = \lim\limits_{x\to\infty} \sec^2(1/x) = 1^2 = 1$

45. $\lim\limits_{x\to 0}\left(\dfrac{1}{x} - \csc x\right) = \lim\limits_{x\to 0}\left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right) = \lim\limits_{x\to 0} \dfrac{\sin x - x}{x\sin x}$

$\overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{\cos x - 1}{x\cos x + \sin x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{-\sin x}{2\cos x - x\sin x} = \dfrac{0}{2} = 0$

46. $\lim\limits_{x\to 0}(\csc x - \cot x) = \lim\limits_{x\to 0}\left(\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}\right) = \lim\limits_{x\to 0} \dfrac{1-\cos x}{\sin x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{\sin x}{\cos x} = 0$

47. We will multiply and divide by the conjugate of the expression to change the form of the expression.

$$\lim_{x\to\infty}\left(\sqrt{x^2+x} - x\right) = \lim_{x\to\infty}\left(\frac{\sqrt{x^2+x}-x}{1}\cdot\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}\right) = \lim_{x\to\infty}\frac{(x^2+x)-x^2}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty}\frac{x}{\sqrt{x^2+x}+x} = \lim_{x\to\infty}\frac{1}{\sqrt{1+1/x}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}.$$

As an alternate solution, write $\sqrt{x^2+x} - x$ as $\sqrt{x^2+x} - \sqrt{x^2}$, factor out $\sqrt{x^2}$, rewrite as $\left(\sqrt{1+1/x} - 1\right)/(1/x)$, and apply l'Hospital's Rule.

48. $\lim\limits_{x\to 1}\left(\dfrac{1}{\ln x} - \dfrac{1}{x-1}\right) = \lim\limits_{x\to 1}\dfrac{x-1-\ln x}{(x-1)\ln x} \overset{\text{H}}{=} \lim\limits_{x\to 1}\dfrac{1-1/x}{(x-1)(1/x)+\ln x}\cdot\dfrac{x}{x}$

$= \lim\limits_{x\to 1}\dfrac{x-1}{x-1+x\ln x} \overset{\text{H}}{=} \lim\limits_{x\to 1}\dfrac{1}{1+1+\ln x} = \dfrac{1}{2+0} = \dfrac{1}{2}$

49. The limit has the form $\infty - \infty$ and we will change the form to a product by factoring out x.

$\lim\limits_{x\to\infty}(x - \ln x) = \lim\limits_{x\to\infty} x\left(1 - \dfrac{\ln x}{x}\right) = \infty$ since $\lim\limits_{x\to\infty}\dfrac{\ln x}{x} \overset{\text{H}}{=} \lim\limits_{x\to\infty}\dfrac{1/x}{1} = 0.$

50. As $x\to\infty$, $1/x\to 0$, and $e^{1/x}\to 1$. So the limit has the form $\infty - \infty$ and we will change the form to a product by factoring out x.

$\lim\limits_{x\to\infty}\left(xe^{1/x} - x\right) = \lim\limits_{x\to\infty} x\left(e^{1/x} - 1\right) = \lim\limits_{x\to\infty}\dfrac{e^{1/x}-1}{1/x} \overset{\text{H}}{=} \lim\limits_{x\to\infty}\dfrac{e^{1/x}(-1/x^2)}{-1/x^2} = \lim\limits_{x\to\infty} e^{1/x} = e^0 = 1$

51. $y = x^{x^2} \quad\Rightarrow\quad \ln y = x^2\ln x$, so

$\lim\limits_{x\to 0^+}\ln y = \lim\limits_{x\to 0^+} x^2\ln x = \lim\limits_{x\to 0^+}\dfrac{\ln x}{1/x^2} \overset{\text{H}}{=} \lim\limits_{x\to 0^+}\dfrac{1/x}{-2/x^3} = \lim\limits_{x\to 0^+}\left(-\dfrac{1}{2}x^2\right) = 0 \quad\Rightarrow$

$\lim\limits_{x\to 0^+} x^{x^2} = \lim\limits_{x\to 0^+} e^{\ln y} = e^0 = 1.$

52. $y = (\tan 2x)^x \quad\Rightarrow\quad \ln y = x\cdot\ln\tan 2x$, so

$\lim\limits_{x\to 0^+}\ln y = \lim\limits_{x\to 0^+} x\cdot\ln\tan 2x = \lim\limits_{x\to 0^+}\dfrac{\ln\tan 2x}{1/x}$

$\overset{\text{H}}{=} \lim\limits_{x\to 0^+}\dfrac{(1/\tan 2x)(2\sec^2 2x)}{-1/x^2} = \lim\limits_{x\to 0^+}\dfrac{-2x^2\cos 2x}{\sin 2x\cos^2 2x} = \lim\limits_{x\to 0^+}\dfrac{2x}{\sin 2x}\cdot\lim\limits_{x\to 0^+}\dfrac{-x}{\cos 2x} = 1\cdot 0 = 0 \quad\Rightarrow$

$\lim\limits_{x\to 0^+}(\tan 2x)^x = \lim\limits_{x\to 0^+} e^{\ln y} = e^0 = 1.$

53. $y = (1-2x)^{1/x} \quad\Rightarrow\quad \ln y = \dfrac{1}{x}\ln(1-2x)$, so $\lim\limits_{x\to 0}\ln y = \lim\limits_{x\to 0}\dfrac{\ln(1-2x)}{x} \overset{\text{H}}{=} \lim\limits_{x\to 0}\dfrac{-2/(1-2x)}{1} = -2 \quad\Rightarrow$

$\lim\limits_{x\to 0}(1-2x)^{1/x} = \lim\limits_{x\to 0} e^{\ln y} = e^{-2}.$