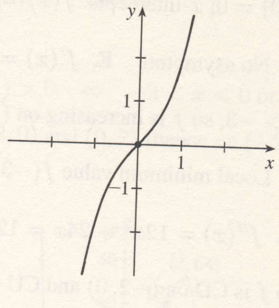
4.5 Summary of Curve Sketching

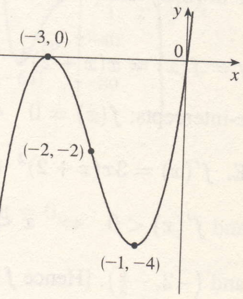
1. $y = f(x) = x^3 + x = x(x^2+1)$ **A.** f is a polynomial, so $D = \mathbb{R}$. **B.** x-intercept $= 0$, y-intercept $= f(0) = 0$ **C.** $f(-x) = -f(x)$, so f is odd; the curve is symmetric about the origin. **D.** f is a polynomial, so there is no asymptote. **E.** $f'(x) = 3x^2 + 1 > 0$, so f is increasing on $(-\infty, \infty)$. **F.** There is no critical number and hence, no local maximum or minimum value. **G.** $f''(x) = 6x > 0$ on $(0, \infty)$ and $f''(x) < 0$ on $(-\infty, 0)$, so f is CU on $(0, \infty)$ and CD on $(-\infty, 0)$. Since the concavity changes at $x = 0$, there is an inflection point at $(0, 0)$.

H.



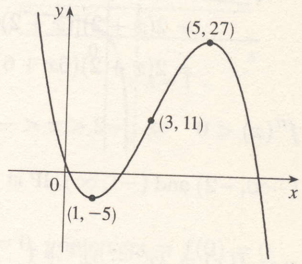
2. $y = f(x) = x^3 + 6x^2 + 9x = x(x+3)^2$ **A.** $D = \mathbb{R}$ **B.** x-intercepts are -3 and 0, y-intercept $= 0$ **C.** No symmetry **D.** No asymptote **E.** $f'(x) = 3x^2 + 12x + 9 = 3(x+1)(x+3) < 0 \;\Leftrightarrow\; -3 < x < -1$, so f is decreasing on $(-3, -1)$ and increasing on $(-\infty, -3)$ and $(-1, \infty)$. **F.** Local maximum value $f(-3) = 0$, local minimum value $f(-1) = -4$ **G.** $f''(x) = 6x + 12 = 6(x+2) > 0 \;\Leftrightarrow\; x > -2$, so f is CU on $(-2, \infty)$ and CD on $(-\infty, -2)$. IP at $(-2, -2)$

H.



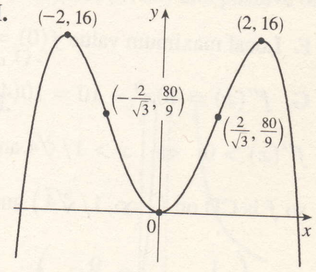
3. $y = f(x) = 2 - 15x + 9x^2 - x^3 = -(x-2)(x^2 - 7x + 1)$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 2$; x-intercepts: $f(x) = 0 \;\Rightarrow\; x = 2$ or (by the quadratic formula) $x = \frac{7 \pm \sqrt{45}}{2} \approx 0.15, 6.85$ **C.** No symmetry **D.** No asymptote
E. $f'(x) = -15 + 18x - 3x^2 = -3(x^2 - 6x + 5)$
$= -3(x-1)(x-5) > 0 \;\Leftrightarrow\; 1 < x < 5$
so f is increasing on $(1, 5)$ and decreasing on $(-\infty, 1)$ and $(5, \infty)$.
F. Local maximum value $f(5) = 27$, local minimum value $f(1) = -5$
G. $f''(x) = 18 - 6x = -6(x-3) > 0 \;\Leftrightarrow\; x < 3$, so f is CU on $(-\infty, 3)$ and CD on $(3, \infty)$. IP at $(3, 11)$

H.



4. $y = f(x) = 8x^2 - x^4 = x^2(8 - x^2)$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \;\Rightarrow\; x = 0, \pm 2\sqrt{2}$ $(\approx \pm 2.83)$ **C.** $f(-x) = f(x)$, so f is even and symmetric about the y-axis. **D.** No asymptote
E. $f'(x) = 16x - 4x^3 = 4x(4 - x^2) = 4x(2+x)(2-x) > 0 \;\Leftrightarrow\; x < -2$ or $0 < x < 2$, so f is increasing on $(-\infty, -2)$ and $(0, 2)$ and decreasing on $(-2, 0)$ and $(2, \infty)$. **F.** Local maximum value $f(\pm 2) = 16$, local minimum value $f(0) = 0$
G. $f''(x) = 16 - 12x^2 = 4(4 - 3x^2) = 0 \;\Leftrightarrow\; x = \pm\frac{2}{\sqrt{3}}$.
$f''(x) > 0 \;\Leftrightarrow\; -\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$, so f is CU on $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ and CD on $\left(-\infty, -\frac{2}{\sqrt{3}}\right)$ and $\left(\frac{2}{\sqrt{3}}, \infty\right)$. IP at $\left(\pm\frac{2}{\sqrt{3}}, \frac{80}{9}\right)$

H.



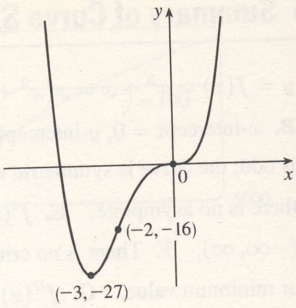
5. $y = f(x) = x^4 + 4x^3 = x^3(x+4)$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \Leftrightarrow x = -4, 0$ **C.** No symmetry **D.** No asymptote **E.** $f'(x) = 4x^3 + 12x^2 = 4x^2(x+3) > 0 \Leftrightarrow x > -3$, so f is increasing on $(-3, \infty)$ and decreasing on $(-\infty, -3)$.

F. Local minimum value $f(-3) = -27$, no local maximum

G. $f''(x) = 12x^2 + 24x = 12x(x+2) < 0 \quad \Leftrightarrow \quad -2 < x < 0$, so f is CD on $(-2, 0)$ and CU on $(-\infty, -2)$ and $(0, \infty)$.

IP at $(0, 0)$ and $(-2, -16)$

H.



6. $y = f(x) = x(x+2)^3$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \quad \Leftrightarrow \quad x = -2, 0$ **C.** No symmetry **D.** No asymptote

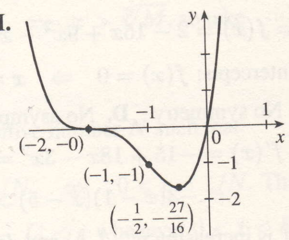
E. $f'(x) = 3x(x+2)^2 + (x+2)^3 = (x+2)^2\,[3x + (x+2)] = (x+2)^2(4x+2)$. $f'(x) > 0 \quad \Leftrightarrow \quad x > -\frac{1}{2}$, and $f'(x) < 0 \quad \Leftrightarrow \quad x < -2$ or $-2 < x < -\frac{1}{2}$, so f is increasing on $\left(-\frac{1}{2}, \infty\right)$ and decreasing on $(-\infty, -2)$ and $\left(-2, -\frac{1}{2}\right)$. [Hence f is decreasing on $\left(-\infty, -\frac{1}{2}\right)$ by the analogue of Exercise 4.3.53 for decreasing functions.]

F. Local minimum value $f\left(-\frac{1}{2}\right) = -\frac{27}{16}$, no local maximum

G.
$$
\begin{aligned}
f''(x) &= (x+2)^2(4) + (4x+2)(2)(x+2) \\
&= 2(x+2)[(x+2)(2) + 4x + 2] \\
&= 2(x+2)(6x+6) = 12(x+1)(x+2)
\end{aligned}
$$

$f''(x) < 0 \quad \Leftrightarrow \quad -2 < x < -1$, so f is CD on $(-2, -1)$ and CU on $(-\infty, -2)$ and $(-1, \infty)$. IP at $(-2, 0)$ and $(-1, -1)$

H.



7. $y = f(x) = 2x^5 - 5x^2 + 1$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 1$ **C.** No symmetry **D.** No asymptote

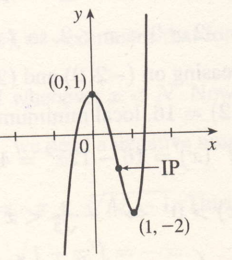
E. $f'(x) = 10x^4 - 10x = 10x(x^3 - 1) = 10x(x-1)(x^2 + x + 1)$, so $f'(x) < 0 \quad \Leftrightarrow \quad 0 < x < 1$ and $f'(x) > 0 \quad \Leftrightarrow \quad x < 0$ or $x > 1$. Thus, f is increasing on $(-\infty, 0)$ and $(1, \infty)$ and decreasing on $(0, 1)$.

F. Local maximum value $f(0) = 1$, local minimum value $f(1) = -2$

G. $f''(x) = 40x^3 - 10 = 10(4x^3 - 1)$ so $f''(x) = 0 \quad \Leftrightarrow \quad x = 1/\sqrt[3]{4}$. $f''(x) > 0 \quad \Leftrightarrow \quad x > 1/\sqrt[3]{4}$ and $f''(x) < 0 \quad \Leftrightarrow \quad x < 1/\sqrt[3]{4}$, so f is CD on $\left(-\infty, 1/\sqrt[3]{4}\right)$ and CU on $\left(1/\sqrt[3]{4}, \infty\right)$.

IP at $\left(\dfrac{1}{\sqrt[3]{4}}, 1 - \dfrac{9}{2\left(\sqrt[3]{4}\right)^2}\right) \approx (0.630, -0.786)$

H.



8. $y = f(x) = 2$
$-3x^3(x^2 - \frac{20}{3}$
the curve is sy
E. $f'(x) = 6$
$0 < x < 2$ and
by Exercise 4.
F. Local mini
value $f(2) =$
$f''(x) > 0$
$-\sqrt{2} < x <$
and $(0, \sqrt{2})$,
$(-\sqrt{2}, -28$

9. $y = f(x) =$
y-intercept $=$
$\lim\limits_{x \to 1^-} \dfrac{x}{x-1}$
E. $f'(x) =$
on $(-\infty, 1)$
G. $f''(x) =$
on $(-\infty, 1)$

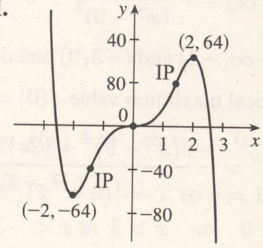
10. $y = x/(x -$
C. No sym
E. $f'(x) =$
$(-1, 1)$, so
F. Local m
G. $f''(x)$
negative or
on $(-\infty, -$

8. $y = f(x) = 20x^3 - 3x^5$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \Leftrightarrow$ $-3x^3\left(x^2 - \frac{20}{3}\right) = 0 \Leftrightarrow x = 0$ or $\pm\sqrt{20/3} \approx \pm 2.582$ **C.** $f(-x) = -f(x)$, so f is odd; the curve is symmetric about the origin. **D.** No asymptote

E. $f'(x) = 60x^2 - 15x^4 = -15x^2(x^2 - 4) = -15x^2(x+2)(x-2)$, so $f'(x) > 0 \Leftrightarrow -2 < x < 0$ or $0 < x < 2$ and $f'(x) < 0 \Leftrightarrow x < -2$ or $x > 2$. Thus, f is increasing on $(-2, 0)$ and $(0, 2)$ [hence on $(-2, 2)$ by Exercise 4.3.53] and f is decreasing on $(-\infty, -2)$ and $(2, \infty)$.

F. Local minimum value $f(-2) = -64$, local maximum value $f(2) = 64$ **G.** $f''(x) = 120x - 60x^3 = -60x(x^2 - 2)$. $f''(x) > 0 \Leftrightarrow x < -\sqrt{2}$ or $0 < x < \sqrt{2}$; $f''(x) < 0 \Leftrightarrow$ $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$. Thus, f is CU on $(-\infty, -\sqrt{2})$ and $(0, \sqrt{2})$, and f is CD on $(-\sqrt{2}, 0)$ and $(\sqrt{2}, \infty)$. IP at $(-\sqrt{2}, -28\sqrt{2}) \approx (-1.414, -39.598)$, $(0, 0)$, and $(\sqrt{2}, 28\sqrt{2})$

H.

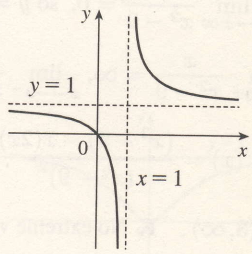


9. $y = f(x) = x/(x-1)$ **A.** $D = \{x \mid x \neq 1\} = (-\infty, 1) \cup (1, \infty)$ **B.** x-intercept $= 0$, y-intercept $= f(0) = 0$ **C.** No symmetry **D.** $\lim\limits_{x \to \pm\infty} \dfrac{x}{x-1} = 1$, so $y = 1$ is a HA. $\lim\limits_{x \to 1^-} \dfrac{x}{x-1} = -\infty$, $\lim\limits_{x \to 1^+} \dfrac{x}{x-1} = \infty$, so $x = 1$ is a VA.

E. $f'(x) = \dfrac{(x-1) - x}{(x-1)^2} = \dfrac{-1}{(x-1)^2} < 0$ for $x \neq 1$, so f is decreasing on $(-\infty, 1)$ and $(1, \infty)$. **F.** No extreme values

G. $f''(x) = \dfrac{2}{(x-1)^3} > 0 \Leftrightarrow x > 1$, so f is CU on $(1, \infty)$ and CD on $(-\infty, 1)$. No IP

H.



10. $y = x/(x-1)^2$ **A.** $D = \{x \mid x \neq 1\} = (-\infty, 1) \cup (1, \infty)$ **B.** x-intercept $= 0$, y-intercept $= f(0) = 0$

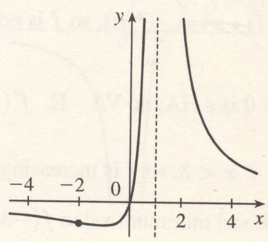
C. No symmetry **D.** $\lim\limits_{x \to \pm\infty} \dfrac{x}{(x-1)^2} = 0$, so $y = 0$ is a HA. $\lim\limits_{x \to 1} \dfrac{x}{(x-1)^2} = \infty$, so $x = 1$ is a VA.

E. $f'(x) = \dfrac{(x-1)^2(1) - x(2)(x-1)}{(x-1)^4} = \dfrac{-x-1}{(x-1)^3}$. This is negative on $(-\infty, -1)$ and $(1, \infty)$ and positive on $(-1, 1)$, so $f(x)$ is decreasing on $(-\infty, -1)$ and $(1, \infty)$ and increasing on $(-1, 1)$.

F. Local minimum value $f(-1) = -\frac{1}{4}$, no local maximum.

G. $f''(x) = \dfrac{(x-1)^3(-1) + (x+1)(3)(x-1)^2}{(x-1)^6} = \dfrac{2(x+2)}{(x-1)^4}$. This is negative on $(-\infty, -2)$, and positive on $(-2, 1)$ and $(1, \infty)$. So f is CD on $(-\infty, -2)$ and CU on $(-2, 1)$ and $(1, \infty)$. IP at $\left(-2, -\frac{2}{9}\right)$

H.



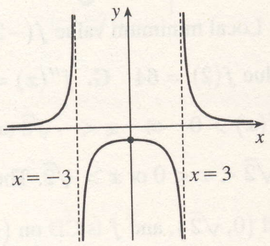
11. $y = f(x) = 1/(x^2 - 9)$ **A.** $D = \{x \mid x \neq \pm 3\} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ **B.** y-intercept $= f(0) = -\frac{1}{9}$, no x-intercept **C.** $f(-x) = f(x) \Rightarrow$ f is even; the curve is symmetric about the y-axis. **D.** $\lim\limits_{x\to\pm\infty} \dfrac{1}{x^2-9} = 0$, so $y = 0$ is a HA. $\lim\limits_{x\to 3^-} \dfrac{1}{x^2-9} = -\infty$, $\lim\limits_{x\to 3^+} \dfrac{1}{x^2-9} = \infty$, $\lim\limits_{x\to -3^-} \dfrac{1}{x^2-9} = \infty$, $\lim\limits_{x\to -3^+} \dfrac{1}{x^2-9} = -\infty$, so $x = 3$ and $x = -3$ are VA.

E. $f'(x) = -\dfrac{2x}{(x^2-9)^2} > 0 \Leftrightarrow x < 0$ $(x \neq -3)$ so f is increasing on $(-\infty, -3)$ and $(-3, 0)$ and decreasing on $(0, 3)$ and $(3, \infty)$.

F. Local maximum value $f(0) = -\frac{1}{9}$.

G. $y'' = \dfrac{-2(x^2-9)^2 + (2x)2(x^2-9)(2x)}{(x^2-9)^4} = \dfrac{6(x^2+3)}{(x^2-9)^3} > 0 \Leftrightarrow$ $x^2 > 9 \Leftrightarrow x > 3$ or $x < -3$, so f is CU on $(-\infty, -3)$ and $(3, \infty)$ and CD on $(-3, 3)$. No IP

H.



12. $y = f(x) = x/(x^2 - 9)$ **A.** $D = \{x \mid x \neq \pm 3\} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ **B.** x-intercept $= 0$, y-intercept $= f(0) = 0$. **C.** $f(-x) = -f(x)$, so f is odd; the curve is symmetric about the origin.

D. $\lim\limits_{x\to\pm\infty} \dfrac{x}{x^2-9} = 0$, so $y = 0$ is a HA. $\lim\limits_{x\to 3^+} \dfrac{x}{x^2-9} = \infty$, $\lim\limits_{x\to 3^-} \dfrac{x}{x^2-9} = -\infty$, $\lim\limits_{x\to -3^+} \dfrac{x}{x^2-9} = \infty$, $\lim\limits_{x\to -3^-} \dfrac{x}{x^2-9} = -\infty$, so $x = 3$ and $x = -3$ are VA.

E. $f'(x) = \dfrac{(x^2-9) - x(2x)}{(x^2-9)^2} = -\dfrac{x^2+9}{(x^2-9)^2} < 0$ $(x \neq \pm 3)$ so f is decreasing on $(-\infty, -3)$, $(-3, 3)$, and $(3, \infty)$. **F.** No extreme values

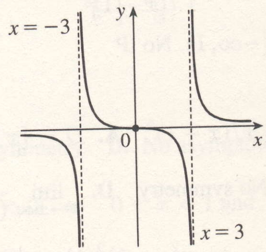
G. $f''(x) = -\dfrac{2x(x^2-9)^2 - (x^2+9)\cdot 2(x^2-9)(2x)}{(x^2-9)^4}$

$= \dfrac{2x(x^2+27)}{(x^2-9)^3} > 0$ when $-3 < x < 0$ or $x > 3$,

so f is CU on $(-3, 0)$ and $(3, \infty)$; CD on $(-\infty, -3)$ and $(0, 3)$.

IP at $(0, 0)$

H.



13. $y = f(x) = x/(x^2 + 9)$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercept: $f(x) = 0 \Leftrightarrow x = 0$ **C.** $f(-x) = -f(x)$, so f is odd and the curve is symmetric about the origin. **D.** $\lim\limits_{x\to\pm\infty} [x/(x^2+9)] = 0$, so $y = 0$ is a HA; no VA **E.** $f'(x) = \dfrac{(x^2+9)(1) - x(2x)}{(x^2+9)^2} = \dfrac{9-x^2}{(x^2+9)^2} = \dfrac{(3+x)(3-x)}{(x^2+9)^2} > 0 \Leftrightarrow$ $-3 < x < 3$, so f is increasing on $(-3, 3)$ and decreasing on $(-\infty, -3)$ and $(3, \infty)$.

F. Local minimum value $f(-3) = -\frac{1}{6}$, local maximum value $f(3) = \frac{1}{6}$

G. $f''(x) = \dfrac{(x^2+9)^2(-2x) - (9-x^2)\cdot 2(x^2+9)(2x)}{[(x^2+9)^2]^2}$

$= \dfrac{(2x)(x^2+9)\left[-(x^2+9) - 2(9-x^2)\right]}{(x^2+9)^4}$

$= \dfrac{2x(x^2-27)}{(x^2+9)^3} = 0 \quad\Leftrightarrow\quad x = 0, \pm\sqrt{27} = \pm 3\sqrt{3}$

$f''(x) > 0 \quad\Leftrightarrow\quad -3\sqrt{3} < x < 0$ or $x > 3\sqrt{3}$, so f is CU on $(-3\sqrt{3}, 0)$ and $(3\sqrt{3}, \infty)$, and CD on $(-\infty, -3\sqrt{3})$ and $(0, 3\sqrt{3})$.
There are three inflection points: $(0,0)$ and $\left(\pm 3\sqrt{3}, \pm\frac{1}{12}\sqrt{3}\right)$.

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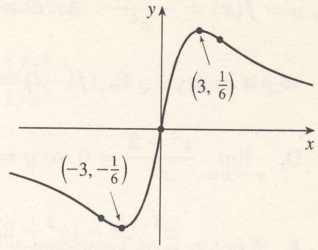


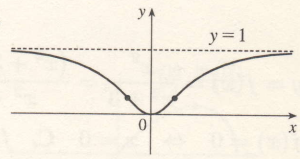
14. $y = f(x) = x^2/(x^2+9)$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercept: $f(x) = 0 \quad\Leftrightarrow\quad x = 0$
C. $f(-x) = f(x)$, so f is even and symmetric about the y-axis. **D.** $\lim\limits_{x\to\pm\infty}\left[x^2/(x^2+9)\right] = 1$, so $y = 1$ is a HA; no VA **E.** $f'(x) = \dfrac{(x^2+9)(2x) - x^2(2x)}{(x^2+9)^2} = \dfrac{18x}{(x^2+9)^2} > 0 \quad\Leftrightarrow\quad x > 0$, so f is increasing on $(0,\infty)$ and decreasing on $(-\infty, 0)$. **F.** Local minimum value $f(0) = 0$; no local maximum

G. $f''(x) = \dfrac{(x^2+9)^2(18) - 18x\cdot 2(x^2+9)\cdot 2x}{[(x^2+9)^2]^2} = \dfrac{18(x^2+9)\left[(x^2+9) - 4x^2\right]}{(x^2+9)^4} = \dfrac{18(9-3x^2)}{(x^2+9)^3}$

$= \dfrac{-54(x+\sqrt{3})(x-\sqrt{3})}{(x^2+9)^3} > 0 \quad\Leftrightarrow\quad -\sqrt{3} < x < \sqrt{3}$

so f is CU on $(-\sqrt{3}, \sqrt{3})$ and CD on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$.
There are two inflection points: $\left(\pm\sqrt{3}, \frac{1}{4}\right)$.

H.



15. $y = f(x) = \dfrac{x-1}{x^2}$ **A.** $D = \{x \mid x \neq 0\} = (-\infty, 0)\cup(0,\infty)$ **B.** No y-intercept; x-intercept: $f(x) = 0 \quad\Leftrightarrow\quad x = 1$ **C.** No symmetry **D.** $\lim\limits_{x\to\pm\infty}\dfrac{x-1}{x^2} = 0$, so $y = 0$ is a HA. $\lim\limits_{x\to 0}\dfrac{x-1}{x^2} = -\infty$, so $x = 0$ is a VA.

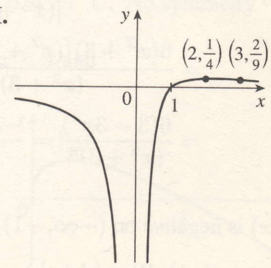
E. $f'(x) = \dfrac{x^2\cdot 1 - (x-1)\cdot 2x}{(x^2)^2} = \dfrac{-x^2+2x}{x^4} = \dfrac{-(x-2)}{x^3}$, so $f'(x) > 0 \quad\Leftrightarrow\quad 0 < x < 2$ and $f'(x) < 0 \quad\Leftrightarrow\quad x < 0$ or $x > 2$. Thus, f is increasing on $(0,2)$ and decreasing on $(-\infty, 0)$ and $(2,\infty)$.

F. No local minimum, local maximum value $f(2) = \frac{1}{4}$.

G. $f''(x) = \dfrac{x^3\cdot(-1) - [-(x-2)]\cdot 3x^2}{(x^3)^2} = \dfrac{2x^3 - 6x^2}{x^6} = \dfrac{2(x-3)}{x^4}$.

$f''(x)$ is negative on $(-\infty, 0)$ and $(0,3)$ and positive on $(3,\infty)$, so f is CD on $(-\infty, 0)$ and $(0,3)$ and CU on $(3,\infty)$. IP at $\left(3, \frac{2}{9}\right)$

H.



16. $y = f(x) = \dfrac{x^2 - 2}{x^4}$ **A.** $D = \{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$ **B.** No y-intercept; x-intercepts: $f(x) = 0$ $\Leftrightarrow$ $x = \pm\sqrt{2}$ **C.** $f(-x) = f(x)$, so f is even; the curve is symmetric about the y-axis.

D. $\displaystyle\lim_{x\to\pm\infty} \frac{x^2 - 2}{x^4} = 0$, so $y = 0$ is a HA. $\displaystyle\lim_{x\to 0} \frac{x^2 - 2}{x^4} = -\infty$, so $x = 0$ is a VA.

E. $f'(x) = \dfrac{x^4 \cdot 2x - (x^2 - 2)(4x^3)}{(x^4)^2} = \dfrac{-2x^5 + 8x^3}{x^8} = \dfrac{-2(x^2 - 4)}{x^5} = \dfrac{-2(x + 2)(x - 2)}{x^5}$.

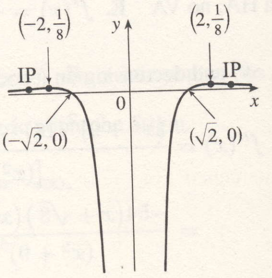
$f'(x)$ is negative on $(-2, 0)$ and $(2, \infty)$ and positive on $(-\infty, -2)$ and $(0, 2)$, so f is decreasing on $(-2, 0)$ and $(2, \infty)$ and increasing on $(-\infty, -2)$ and $(0, 2)$. **F.** Local maximum value $f(\pm 2) = \frac{1}{8}$, no local minimum.

G. $f''(x) = \dfrac{x^5 \cdot (-4x) + 2(x^2 - 4) \cdot 5x^4}{(x^5)^2} = \dfrac{2x^4[-2x^2 + 5(x^2 - 4)]}{x^{10}} = \dfrac{2(3x^2 - 20)}{x^6}$

$f''(x)$ is positive on $\left(-\infty, -\sqrt{\frac{20}{3}}\right)$ and $\left(\sqrt{\frac{20}{3}}, \infty\right)$ and negative on $\left(-\sqrt{\frac{20}{3}}, 0\right)$ and $\left(0, \sqrt{\frac{20}{3}}\right)$, so f is CU on $\left(-\infty, -\sqrt{\frac{20}{3}}\right)$ and $\left(\sqrt{\frac{20}{3}}, \infty\right)$ and CD on $\left(-\sqrt{\frac{20}{3}}, 0\right)$ and $\left(0, \sqrt{\frac{20}{3}}\right)$.

IP at $\left(\pm\sqrt{\frac{20}{3}}, \frac{21}{200}\right) \approx (\pm 2.5820, 0.105)$

H.



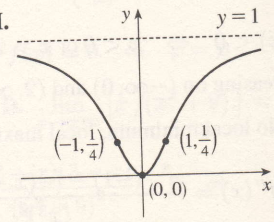
17. $y = f(x) = \dfrac{x^2}{x^2 + 3} = \dfrac{(x^2 + 3) - 3}{x^2 + 3} = 1 - \dfrac{3}{x^2 + 3}$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0$ $\Leftrightarrow$ $x = 0$ **C.** $f(-x) = f(x)$, so f is even; the graph is symmetric about the y-axis.

D. $\displaystyle\lim_{x\to\pm\infty} \frac{x^2}{x^2 + 3} = 1$, so $y = 1$ is a HA. No VA. **E.** Using the Reciprocal Rule,

$f'(x) = -3 \cdot \dfrac{-2x}{(x^2 + 3)^2} = \dfrac{6x}{(x^2 + 3)^2}$. $f'(x) > 0$ $\Leftrightarrow$ $x > 0$ and $f'(x) < 0$ $\Leftrightarrow$ $x < 0$, so f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$. **F.** Local minimum value $f(0) = 0$, no local maximum.

G.
$$f''(x) = \frac{(x^2 + 3)^2 \cdot 6 - 6x \cdot 2(x^2 + 3) \cdot 2x}{[(x^2 + 3)^2]^2} = \frac{6(x^2 + 3)[(x^2 + 3) - 4x^2]}{(x^2 + 3)^4} = \frac{6(3 - 3x^2)}{(x^2 + 3)^3} = \frac{-18(x + 1)(x - 1)}{(x^2 + 3)^3}$$

H.



$f''(x)$ is negative on $(-\infty, -1)$ and $(1, \infty)$ and positive on $(-1, 1)$, so f is CD on $(-\infty, -1)$ and $(1, \infty)$ and CU on $(-1, 1)$. IP at $(\pm 1, \frac{1}{4})$

18. $y = f(x) = \dfrac{x^3 - 1}{x^3 + 1}$ **A.** $D = \{x \mid x \neq -1\} = (-\infty, -1) \cup (-1, \infty)$ **B.** x-intercept $= 1$, y-intercept $= f(0) = -1$ **C.** No symmetry **D.** $\displaystyle\lim_{x \to \pm\infty} \frac{x^3 - 1}{x^3 + 1} = \lim_{x \to \pm\infty} \frac{1 - 1/x^3}{1 + 1/x^3} = 1$, so $y = 1$ is a HA. $\displaystyle\lim_{x \to -1^-} \frac{x^3 - 1}{x^3 + 1} = \infty$ and $\displaystyle\lim_{x \to -1^+} \frac{x^3 - 1}{x^3 + 1} = -\infty$, so $x = -1$ is a VA.

E. $f'(x) = \dfrac{(x^3 + 1)(3x^2) - (x^3 - 1)(3x^2)}{(x^3 + 1)^2} = \dfrac{6x^2}{(x^3 + 1)^2} > 0$ $(x \neq -1)$ so f is increasing on $(-\infty, -1)$ and $(-1, \infty)$. **F.** No extreme values

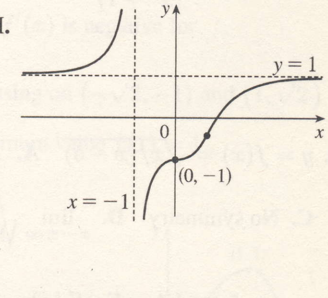
G. $y'' = \dfrac{12x(x^3 + 1)^2 - 6x^2 \cdot 2(x^3 + 1) \cdot 3x^2}{(x^3 + 1)^4}$

$= \dfrac{12x(1 - 2x^3)}{(x^3 + 1)^3} > 0 \quad \Leftrightarrow \quad x < -1 \text{ or } 0 < x < \dfrac{1}{\sqrt[3]{2}}$,

so f is CU on $(-\infty, -1)$ and $\left(0, \frac{1}{\sqrt[3]{2}}\right)$ and CD on $(-1, 0)$ and $\left(\frac{1}{\sqrt[3]{2}}, \infty\right)$.

IP at $(0, -1)$, $\left(\frac{1}{\sqrt[3]{2}}, -\frac{1}{3}\right)$

H.



19. $y = f(x) = x\sqrt{5 - x}$ **A.** The domain is $\{x \mid 5 - x \geq 0\} = (-\infty, 5]$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \quad \Leftrightarrow \quad x = 0, 5$ **C.** No symmetry **D.** No asymptote

E. $f'(x) = x \cdot \frac{1}{2}(5 - x)^{-1/2}(-1) + (5 - x)^{1/2} \cdot 1 = \frac{1}{2}(5 - x)^{-1/2}[-x + 2(5 - x)] = \dfrac{10 - 3x}{2\sqrt{5 - x}} > 0 \quad \Leftrightarrow$ $x < \frac{10}{3}$, so f is increasing on $\left(-\infty, \frac{10}{3}\right)$ and decreasing on $\left(\frac{10}{3}, 5\right)$.

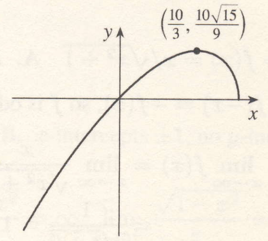
F. Local maximum value $f\left(\frac{10}{3}\right) = \frac{10}{9}\sqrt{15} \approx 4.3$; no local minimum

G. $f''(x) = \dfrac{2(5 - x)^{1/2}(-3) - (10 - 3x) \cdot 2\left(\frac{1}{2}\right)(5 - x)^{-1/2}(-1)}{\left(2\sqrt{5 - x}\right)^2}$

$= \dfrac{(5 - x)^{-1/2}[-6(5 - x) + (10 - 3x)]}{4(5 - x)} = \dfrac{3x - 20}{4(5 - x)^{3/2}}$

$f''(x) < 0$ for $x < 5$, so f is CD on $(-\infty, 5)$. No IP

H.



20. $y = f(x) = 2\sqrt{x} - x$ **A.** $D = [0, \infty)$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \quad \Rightarrow$ $2\sqrt{x} = x \quad \Rightarrow \quad 4x = x^2 \quad \Rightarrow \quad 4x - x^2 = 0 \quad \Rightarrow \quad x(4 - x) = 0 \quad \Rightarrow \quad x = 0, 4$ **C.** No symmetry

D. No asymptote **E.** $f'(x) = \dfrac{1}{\sqrt{x}} - 1 = \dfrac{1}{\sqrt{x}}(1 - \sqrt{x})$. This is positive for $x < 1$ and negative for $x > 1$, so f is increasing on $(0, 1)$ and decreasing on $(1, \infty)$.

F. Local maximum value $f(1) = 1$, no local minimum.

G. $f''(x) = \left(x^{-1/2} - 1\right)' = -\frac{1}{2}x^{-3/2} = \dfrac{-1}{2x^{3/2}} < 0$ for $x > 0$, so f is CD on $(0, \infty)$. No IP

H.