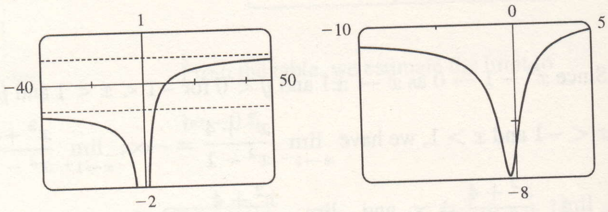
40. $\displaystyle\lim_{x\to\infty}\frac{x-9}{\sqrt{4x^2+3x+2}}=\lim_{x\to\infty}\frac{1-9/x}{\sqrt{4+(3/x)+(2/x^2)}}=\frac{1-0}{\sqrt{4+0+0}}=\frac{1}{2}.$

Using the fact that $\sqrt{x^2}=|x|=-x$ for $x<0$, we divide the numerator by $-x$ and the denominator by $\sqrt{x^2}$.

Thus, $\displaystyle\lim_{x\to-\infty}\frac{x-9}{\sqrt{4x^2+3x+2}}=\lim_{x\to-\infty}\frac{-1+9/x}{\sqrt{4+(3/x)+(2/x^2)}}=\frac{-1+0}{\sqrt{4+0+0}}=-\frac{1}{2}.$

The horizontal asymptotes are $y=\pm\frac{1}{2}$. The polynomial $4x^2+3x+2$ is positive for all x, so the denominator never approaches zero, and thus there is no vertical asymptote.



41. Let's look for a rational function.

(1) $\displaystyle\lim_{x\to\pm\infty}f(x)=0\ \Rightarrow$ degree of numerator $<$ degree of denominator

(2) $\displaystyle\lim_{x\to0}f(x)=-\infty\ \Rightarrow$ there is a factor of x^2 in the denominator (not just x, since that would produce a sign change at $x=0$), and the function is negative near $x=0$.

(3) $\displaystyle\lim_{x\to3^-}f(x)=\infty$ and $\displaystyle\lim_{x\to3^+}f(x)=-\infty\ \Rightarrow$ vertical asymptote at $x=3$; there is a factor of $(x-3)$ in the denominator.

(4) $f(2)=0\ \Rightarrow$ 2 is an x-intercept; there is at least one factor of $(x-2)$ in the numerator.

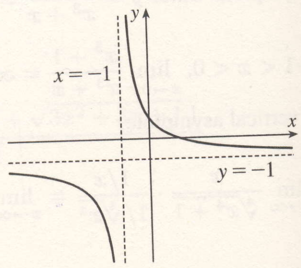
Combining all of this information and putting in a negative sign to give us the desired left- and right-hand limits gives us $f(x)=\dfrac{2-x}{x^2(x-3)}$ as one possibility.

42. Since the function has vertical asymptotes $x=1$ and $x=3$, the denominator of the rational function we are looking for must have factors $(x-1)$ and $(x-3)$. Because the horizontal asymptote is $y=1$, the degree of the numerator must equal the degree of the denominator, and the ratio of the leading coefficients must be 1. One possibility is $f(x)=\dfrac{x^2}{(x-1)(x-3)}$.

43. $y=\dfrac{1-x}{1+x}$ has domain $(-\infty,-1)\cup(-1,\infty)$.

$\displaystyle\lim_{x\to\pm\infty}\frac{1-x}{1+x}=\lim_{x\to\pm\infty}\frac{1/x-1}{1/x+1}=\frac{0-1}{0+1}=-1$, so $y=-1$ is a HA.

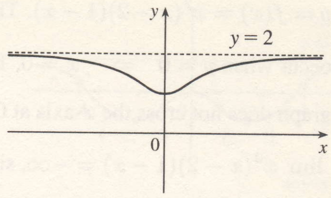
The line $x=-1$ is a VA.



$y'=\dfrac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}=\dfrac{-2}{(1+x)^2}<0$ for $x\neq1$. Thus, $(-\infty,-1)$ and $(-1,\infty)$ are intervals of decrease.

$y''=-2\cdot\dfrac{-2(1+x)}{[(1+x)^2]^2}=\dfrac{4}{(1+x)^3}<0$ for $x<-1$ and $y''>0$ for $x>-1$, so the curve is CD on $(-\infty,-1)$ and CU on $(-1,\infty)$. Since $x=-1$ is not in the domain, there is no IP.

44. $y = \dfrac{1+2x^2}{1+x^2}$ has domain $\mathbb{R}$.



$\displaystyle\lim_{x\to\pm\infty}\frac{1+2x^2}{1+x^2} = \lim_{x\to\pm\infty}\frac{1/x^2+2}{1/x^2+1} = \frac{0+2}{0+1} = 2$, so $y=2$ is a HA.

There is no VA. $y' = \dfrac{(1+x^2)(4x)-(1+2x^2)(2x)}{(1+x^2)^2} = \dfrac{2x}{(1+x^2)^2} > 0$

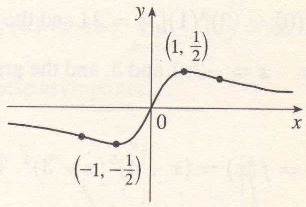
$\Leftrightarrow \;\; x > 0$,

and $y' < 0 \;\;\Leftrightarrow\;\; x < 0$. Thus, y is increasing on $(0,\infty)$ and y is decreasing on $(-\infty,0)$. There is a local (and absolute) minimum at $(0,1)$. $y'' = \dfrac{(1+x^2)^2(2)-(2x)\cdot 2(1+x^2)(2x)}{\left[(1+x^2)^2\right]^2} = \dfrac{2-6x^2}{(1+x^2)^3} = 0 \;\;\Leftrightarrow\;\; x = \pm\frac{1}{\sqrt{3}}$.

$y'' > 0 \;\;\Leftrightarrow\;\; -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, so the curve is CU on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ and CD on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$.

There are IP at $\left(\pm\frac{1}{\sqrt{3}}, \frac{5}{4}\right)$.

45. $\displaystyle\lim_{x\to\pm\infty}\frac{x}{x^2+1} = \lim_{x\to\pm\infty}\frac{1/x}{1+1/x^2} = \frac{0}{1+0} = 0$, so $y=0$ is a horizontal asymptote.

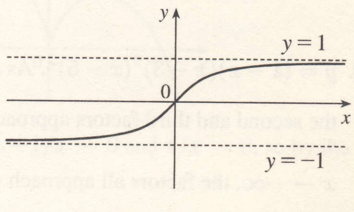


$y' = \dfrac{x^2+1-x(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2} = 0$ when $x = \pm 1$ and $y' > 0 \;\;\Leftrightarrow$

$x^2 < 1 \;\;\Leftrightarrow\;\; -1 < x < 1$, so y is increasing on $(-1,1)$ and decreasing on $(-\infty,-1)$ and $(1,\infty)$.

$y'' = \dfrac{(1+x^2)^2(-2x)-(1-x^2)2(x^2+1)2x}{(1+x^2)^4} = \dfrac{2x(x^2-3)}{(1+x^2)^3} > 0 \;\;\Leftrightarrow\;\; x > \sqrt{3}$ or $-\sqrt{3} < x < 0$, so y is CU on $(\sqrt{3},\infty)$ and $(-\sqrt{3},0)$ and CD on $(-\infty,-\sqrt{3})$ and $(0,\sqrt{3})$.

46. $y = \dfrac{x}{\sqrt{x^2+1}} = \dfrac{x/|x|}{\sqrt{1+1/x^2}}$ has domain $\mathbb{R}$. As $x\to\pm\infty$, $y\to\pm1$, so $y=\pm1$ are HA. There is no VA. $y = x\left(x^2+1\right)^{-1/2} \;\;\Rightarrow$



$$\begin{aligned} y' &= x\left(-\tfrac{1}{2}\right)\left(x^2+1\right)^{-3/2}(2x) + \left(x^2+1\right)^{-1/2}(1) \\ &= \left(x^2+1\right)^{-3/2}\left[-x^2+\left(x^2+1\right)\right] \\ &= \left(x^2+1\right)^{-3/2} > 0 \text{ for all } x \end{aligned}$$

Thus, y is increasing for all x. $y'' = \left(-\frac{3}{2}\right)\left(x^2+1\right)^{-5/2}(2x) = \dfrac{-3x}{(x^2+1)^{5/2}} > 0$ for $x < 0$. So the curve is CU on $(-\infty,0)$ and CD on $(0,\infty)$. There is an inflection point at $(0,0)$.