12.9 Exercises

1. If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 10, what is the radius of convergence of the series $\sum_{n=1}^{\infty} nc_n x^{n-1}$? Why?

2. Suppose you know that the series $\sum_{n=0}^{\infty} b_n x^n$ converges for $|x| < 2$. What can you say about the following series? Why?

$$\sum_{n=0}^{\infty} \frac{b_n}{n+1} x^{n+1}$$

3–10 Find a power series representation for the function and determine the interval of convergence.

3. $f(x) = \dfrac{1}{1+x}$ **4.** $f(x) = \dfrac{3}{1-x^4}$

5. $f(x) = \dfrac{1}{1-x^3}$ **6.** $f(x) = \dfrac{1}{1+9x^2}$

7. $f(x) = \dfrac{1}{x-5}$ **8.** $f(x) = \dfrac{x}{4x+1}$

9. $f(x) = \dfrac{x}{9+x^2}$ **10.** $f(x) = \dfrac{x^2}{a^3 - x^3}$

11–12 Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

11. $f(x) = \dfrac{3}{x^2 + x - 2}$

12. $f(x) = \dfrac{7x-1}{3x^2 + 2x - 1}$

13. (a) Use differentiation to find a power series representation for

$$f(x) = \frac{1}{(1+x)^2}$$

What is the radius of convergence?

(b) Use part (a) to find a power series for

$$f(x) = \frac{1}{(1+x)^3}$$

(c) Use part (b) to find a power series for

$$f(x) = \frac{x^2}{(1+x)^3}$$

14. (a) Find a power series representation for $f(x) = \ln(1+x)$. What is the radius of convergence?

(b) Use part (a) to find a power series for $f(x) = x\ln(1+x)$.

(c) Use part (a) to find a power series for $f(x) = \ln(x^2+1)$.

15–18 Find a power series representation for the function and determine the radius of convergence.

15. $f(x) = \ln(5-x)$ **16.** $f(x) = \dfrac{x^2}{(1-2x)^2}$

17. $f(x) = \dfrac{x^3}{(x-2)^2}$ **18.** $f(x) = \arctan(x/3)$

19–22 Find a power series representation for f, and graph f and several partial sums $s_n(x)$ on the same screen. What happens as n increases?

19. $f(x) = \ln(3+x)$ **20.** $f(x) = \dfrac{1}{x^2+25}$

21. $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$ **22.** $f(x) = \tan^{-1}(2x)$

23–26 Evaluate the indefinite integral as a power series. What is the radius of convergence?

23. $\displaystyle\int \frac{t}{1-t^8}\,dt$ **24.** $\displaystyle\int \frac{\ln(1-t)}{t}\,dt$

25. $\displaystyle\int \frac{x - \tan^{-1}x}{x^3}\,dx$ **26.** $\displaystyle\int \tan^{-1}(x^2)\,dx$

27–30 Use a power series to approximate the definite integral to six decimal places.

27. $\displaystyle\int_0^{0.2} \frac{1}{1+x^5}\,dx$ **28.** $\displaystyle\int_0^{0.4} \ln(1+x^4)\,dx$

29. $\displaystyle\int_0^{1/3} x^2 \tan^{-1}(x^4)\,dx$ **30.** $\displaystyle\int_0^{0.5} \frac{dx}{1+x^6}$

31. Use the result of Example 6 to compute ln 1.1 correct to five decimal places.

32. Show that the function

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

is a solution of the differential equation

$$f''(x) + f(x) = 0$$

33. (a) Show that J_0 (the Bessel function of order 0 given in Example 4) satisfies the differential equation

$$x^2 J_0''(x) + xJ_0'(x) + x^2 J_0(x) = 0$$

(b) Evaluate $\int_0^1 J_0(x)\,dx$ correct to three decimal places.