The inequality $|x + 2| < 3$ can be written as $-5 < x < 1$, so we test the series at the endpoints -5 and 1. When $x = -5$, the series is

$$\sum_{n=0}^{\infty} \frac{n(-3)^n}{3^{n+1}} = \tfrac{1}{3} \sum_{n=0}^{\infty} (-1)^n n$$

which diverges by the Test for Divergence [$(-1)^n n$ doesn't converge to 0]. When $x = 1$, the series is

$$\sum_{n=0}^{\infty} \frac{n(3)^n}{3^{n+1}} = \tfrac{1}{3} \sum_{n=0}^{\infty} n$$

which also diverges by the Test for Divergence. Thus, the series converges only when $-5 < x < 1$, so the interval of convergence is $(-5, 1)$.

12.8 Exercises

1. What is a power series?

2. (a) What is the radius of convergence of a power series? How do you find it?
   (b) What is the interval of convergence of a power series? How do you find it?

3–28 Find the radius of convergence and interval of convergence of the series.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

4. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3}$

6. $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}\, x^n$

7. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

8. $\displaystyle\sum_{n=1}^{\infty} n^n x^n$

9. $\displaystyle\sum_{n=1}^{\infty} (-1)^n n 4^n x^n$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n 3^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{5^n n^5}$

13. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$

14. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

15. $\displaystyle\sum_{n=0}^{\infty} \sqrt{n}\,(x-1)^n$

16. $\displaystyle\sum_{n=0}^{\infty} n^3 (x-5)^n$

17. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n 2^n}$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n 3^n}$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{b^n}(x-a)^n, \quad b > 0$

22. $\displaystyle\sum_{n=1}^{\infty} \frac{n(x-4)^n}{n^3+1}$

23. $\displaystyle\sum_{n=1}^{\infty} n!(2x-1)^n$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}$

25. $\displaystyle\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$

26. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{(2x+3)^n}{n \ln n}$

27. $\displaystyle\sum_{n=2}^{\infty} \frac{x^n}{(\ln n)^n}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)} x^n$

29. If $\sum_{n=0}^{\infty} c_n 4^n$ is convergent, does it follow that the following series are convergent?
    (a) $\displaystyle\sum_{n=0}^{\infty} c_n(-2)^n$ (b) $\displaystyle\sum_{n=0}^{\infty} c_n(-4)^n$

30. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series?
    (a) $\displaystyle\sum_{n=0}^{\infty} c_n$ (b) $\displaystyle\sum_{n=0}^{\infty} c_n 8^n$
    (c) $\displaystyle\sum_{n=0}^{\infty} c_n(-3)^n$ (d) $\displaystyle\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

31. If k is a positive integer, find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$$

32. Graph the first several partial sums $s_n(x)$ of the series $\sum_{n=0}^{\infty} x^n$, together with the sum function $f(x) = 1/(1-x)$, on a common screen. On what interval do these partial sums appear to be converging to $f(x)$?

33. The function J_1 defined by

$$J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$$

is called the *Bessel function of order 1*.
    (a) Find its domain.
    (b) Graph the first several partial sums on a common screen.