NOTE 4 ▫ A finite number of terms doesn't affect the convergence or divergence of a series. For instance, suppose that we were able to show that the series

$$\sum_{n=4}^{\infty} \frac{n}{n^3 + 1}$$

is convergent. Since

$$\sum_{n=1}^{\infty} \frac{n}{n^3 + 1} = \frac{1}{2} + \frac{2}{9} + \frac{3}{28} + \sum_{n=4}^{\infty} \frac{n}{n^3 + 1}$$

it follows that the entire series $\sum_{n=1}^{\infty} n/(n^3 + 1)$ is convergent. Similarly, if it is known that the series $\sum_{n=N+1}^{\infty} a_n$ converges, then the full series

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{N} a_n + \sum_{n=N+1}^{\infty} a_n$$

is also convergent.

|||| **12.2 Exercises**

1. (a) What is the difference between a sequence and a series?
(b) What is a convergent series? What is a divergent series?

2. Explain what it means to say that $\sum_{n=1}^{\infty} a_n = 5$.

3–8 |||| Find at least 10 partial sums of the series. Graph both the sequence of terms and the sequence of partial sums on the same screen. Does it appear that the series is convergent or divergent? If it is convergent, find the sum. If it is divergent, explain why.

3. $\sum_{n=1}^{\infty} \frac{12}{(-5)^n}$

4. $\sum_{n=1}^{\infty} \frac{2n^2 - 1}{n^2 + 1}$

5. $\sum_{n=1}^{\infty} \tan n$

6. $\sum_{n=1}^{\infty} (0.6)^{n-1}$

7. $\sum_{n=1}^{\infty} \left(\frac{1}{n^{1.5}} - \frac{1}{(n + 1)^{1.5}} \right)$

8. $\sum_{n=2}^{\infty} \frac{1}{n(n - 1)}$

9. Let $a_n = \frac{2n}{3n + 1}$.
(a) Determine whether $\{a_n\}$ is convergent.
(b) Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.

10. (a) Explain the difference between

$$\sum_{i=1}^{n} a_i \quad \text{and} \quad \sum_{j=1}^{n} a_j$$

(b) Explain the difference between

$$\sum_{i=1}^{n} a_i \quad \text{and} \quad \sum_{i=1}^{n} a_j$$

11–34 |||| Determine whether the series is convergent or divergent. If it is convergent, find its sum.

11. $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$

12. $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 + \cdots$

13. $-2 + \frac{5}{2} - \frac{25}{8} + \frac{125}{32} - \cdots$

14. $1 + 0.4 + 0.16 + 0.064 + \cdots$

15. $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1}$

16. $\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{5^{n-1}}$

17. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

18. $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

19. $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$

20. $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

21. $\sum_{n=1}^{\infty} \frac{n}{n + 5}$

22. $\sum_{n=1}^{\infty} \frac{3}{n}$

23. $\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$

24. $\sum_{n=1}^{\infty} \frac{(n + 1)^2}{n(n + 2)}$

25. $\sum_{k=2}^{\infty} \frac{k^2}{k^2 - 1}$

26. $\sum_{n=1}^{\infty} \frac{2}{n^2 + 4n + 3}$

27. $\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}$

28. $\sum_{n=1}^{\infty} [(0.8)^{n-1} - (0.3)^n]$

29. $\sum_{n=1}^{\infty} \sqrt[n]{2}$

30. $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n + 5}\right)$

31. $\sum_{n=1}^{\infty} \arctan n$

32. $\sum_{k=1}^{\infty} (\cos 1)^k$

33. $\sum_{n=1}^{\infty} \left(\frac{3}{n(n + 3)} + \frac{5}{4^n} \right)$

34. $\sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n} \right)$

35–40 |||| Express the number as a ratio of integers.

35. $0.\overline{2} = 0.2222\ldots$

36. $0.\overline{73} = 0.73737373\ldots$

37. $3.\overline{417} = 3.417417417\ldots$

38. $6.2\overline{54}$

39. $0.123\overline{456}$

40. $5.6\overline{021}$