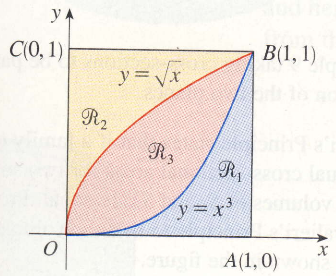
6.2 Exercises

1–18 Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

1. $y = x^2$, $x = 1$, $y = 0$; about the x-axis
2. $x + 2y = 2$, $x = 0$ $y = 0$; about the x-axis
3. $y = 1/x$, $x = 1$, $x = 2$, $y = 0$; about the x-axis
4. $y = \sqrt{x - 1}$, $x = 2$, $x = 5$, $y = 0$; about the x-axis
5. $y = x^2$, $0 \leqslant x \leqslant 2$, $y = 4$, $x = 0$; about the y-axis
6. $x = y - y^2$, $x = 0$; about the y-axis
7. $y = x^2$, $y^2 = x$; about the x-axis
8. $y = \sec x$, $y = 1$, $x = -1$, $x = 1$; about the x-axis
9. $y^2 = x$, $x = 2y$; about the y-axis
10. $y = x^{2/3}$, $x = 1$, $y = 0$; about the y-axis
11. $y = x$, $y = \sqrt{x}$; about $y = 1$
12. $y = x^2$, $y = 4$; about $y = 4$
13. $y = x^4$, $y = 1$; about $y = 2$
14. $y = 1/x^2$, $y = 0$, $x = 1$, $x = 3$; about $y = -1$
15. $x = y^2$, $x = 1$; about $x = 1$
16. $y = x$, $y = \sqrt{x}$; about $x = 2$
17. $y = x^2$, $x = y^2$; about $x = -1$
18. $y = x$, $y = 0$, $x = 2$, $x = 4$; about $x = 1$

19–30 Refer to the figure and find the volume generated by rotating the given region about the specified line.



19. $\mathcal{R}_1$ about OA
20. $\mathcal{R}_1$ about OC
21. $\mathcal{R}_1$ about AB
22. $\mathcal{R}_1$ about BC
23. $\mathcal{R}_2$ about OA
24. $\mathcal{R}_2$ about OC
25. $\mathcal{R}_2$ about AB
26. $\mathcal{R}_2$ about BC
27. $\mathcal{R}_3$ about OA
28. $\mathcal{R}_3$ about OC
29. $\mathcal{R}_3$ about AB
30. $\mathcal{R}_3$ about BC

31–36 Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

31. $y = \tan^3 x$, $y = 1$, $x = 0$; about $y = 1$
32. $y = (x - 2)^4$, $8x - y = 16$; about $x = 10$
33. $y = 0$, $y = \sin x$, $0 \leqslant x \leqslant \pi$; about $y = 1$
34. $y = 0$, $y = \sin x$, $0 \leqslant x \leqslant \pi$; about $y = -2$
35. $x^2 - y^2 = 1$, $x = 3$; about $x = -2$
36. $2x + 3y = 6$, $(y - 1)^2 = 4 - x$; about $x = -5$

37–38 Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the volume of the solid obtained by rotating about the x-axis the region bounded by these curves.

37. $y = x^2$, $y = \sqrt{x + 1}$
38. $y = x^4$, $y = 3x - x^3$

CAS **39–40** Use a computer algebra system to find the exact volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

39. $y = \sin^2 x$, $y = 0$, $0 \leqslant x \leqslant \pi$; about $y = -1$
40. $y = x^2 - 2x$, $y = x\cos(\pi x/4)$; about $y = 2$

41–44 Each integral represents the volume of a solid. Describe the solid.

41. $\pi \int_0^{\pi/2} \cos^2 x\,dx$
42. $\pi \int_2^5 y\,dy$
43. $\pi \int_0^1 (y^4 - y^8)\,dy$
44. $\pi \int_0^{\pi/2} [(1 + \cos x)^2 - 1^2]\,dx$

45. A CAT scan produces equally spaced cross-sectional views of a human organ that provide information about the organ otherwise obtained only by surgery. Suppose that a CAT scan of a human liver shows cross-sections spaced 1.5 cm apart. The liver is 15 cm long and the cross-sectional areas, in square centimeters, are 0, 18, 58, 79, 94, 106, 117, 128, 63, 39, and 0. Use the Midpoint Rule to estimate the volume of the liver.

46. A log 10 m long is cut at 1-meter intervals and its cross-sectional areas A (at a distance x from the end of the log) are