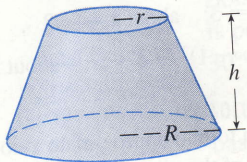


listed in the table. Use the Midpoint Rule with  $n = 5$  to estimate the volume of the log.

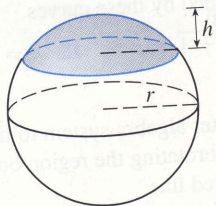
$x$ (m)	$A$ (m <sup>2</sup> )	$x$ (m)	$A$ (m <sup>2</sup> )
0	0.68	6	0.53
1	0.65	7	0.55
2	0.64	8	0.52
3	0.61	9	0.50
4	0.58	10	0.48
5	0.59		

47–59 ||| Find the volume of the described solid  $S$ .

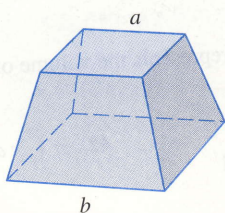
47. A right circular cone with height  $h$  and base radius  $r$
48. A frustum of a right circular cone with height  $h$ , lower base radius  $R$ , and top radius  $r$



49. A cap of a sphere with radius  $r$  and height  $h$

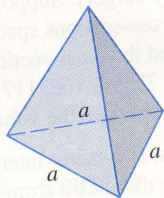


50. A frustum of a pyramid with square base of side  $b$ , square top of side  $a$ , and height  $h$



What happens if  $a = b$ ? What happens if  $a = 0$ ?

51. A pyramid with height  $h$  and rectangular base with dimensions  $b$  and  $2b$
52. A pyramid with height  $h$  and base an equilateral triangle with side  $a$  (a tetrahedron)



53. A tetrahedron with three mutually perpendicular faces and three mutually perpendicular edges with lengths 3 cm, 4 cm, and 5 cm

54. The base of  $S$  is a circular disk with radius  $r$ . Parallel cross-sections perpendicular to the base are squares.

55. The base of  $S$  is an elliptical region with boundary curve  $9x^2 + 4y^2 = 36$ . Cross-sections perpendicular to the  $x$ -axis are isosceles right triangles with hypotenuse in the base.

56. The base of  $S$  is the parabolic region  $\{(x, y) \mid x^2 \leq y \leq 1\}$ . Cross-sections perpendicular to the  $y$ -axis are equilateral triangles.

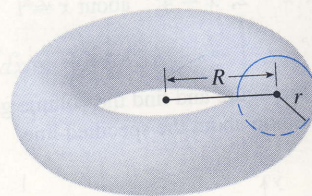
57.  $S$  has the same base as in Exercise 56, but cross-sections perpendicular to the  $y$ -axis are squares.

58. The base of  $S$  is the triangular region with vertices  $(0, 0)$ ,  $(3, 0)$ , and  $(0, 2)$ . Cross-sections perpendicular to the  $y$ -axis are semicircles.

59.  $S$  has the same base as in Exercise 58, but cross-sections perpendicular to the  $y$ -axis are isosceles triangles with height equal to the base.

60. The base of  $S$  is a circular disk with radius  $r$ . Parallel cross-sections perpendicular to the base are isosceles triangles with height  $h$  and unequal side in the base.

- (a) Set up an integral for the volume of  $S$ .
- (b) By interpreting the integral as an area, find the volume of  $S$ .
61. (a) Set up an integral for the volume of a solid torus (the donut-shaped solid shown in the figure) with radii  $r$  and  $R$ .
- (b) By interpreting the integral as an area, find the volume of the torus.



62. Solve Example 9 taking cross-sections to be parallel to the line of intersection of the two planes.

63. (a) Cavalieri's Principle states that if a family of parallel planes gives equal cross-sectional areas for two solids  $S_1$  and  $S_2$ , then the volumes of  $S_1$  and  $S_2$  are equal. Prove this principle.
- (b) Use Cavalieri's Principle to find the volume of the oblique cylinder shown in the figure.

