We must integrate between the appropriate y-values, $y = -2$ and $y = 4$. Thus

$$A = \int_{-2}^{4} (x_R - x_L)\,dy = \int_{-2}^{4} \left[(y+1) - \left(\tfrac{1}{2}y^2 - 3\right)\right] dy$$

$$= \int_{-2}^{4} \left(-\tfrac{1}{2}y^2 + y + 4\right) dy = -\frac{1}{2}\left(\frac{y^3}{3}\right) + \frac{y^2}{2} + 4y\,\Big]_{-2}^{4}$$

$$= -\tfrac{1}{6}(64) + 8 + 16 - \left(\tfrac{4}{3} + 2 - 8\right) = 18$$

We could have found the area in Example 6 by integrating with respect to x instead of y, but the calculation is much more involved. It would have meant splitting the region in two and computing the areas labeled A_1 and A_2 in Figure 14. The method we used in Example 6 is *much* easier.

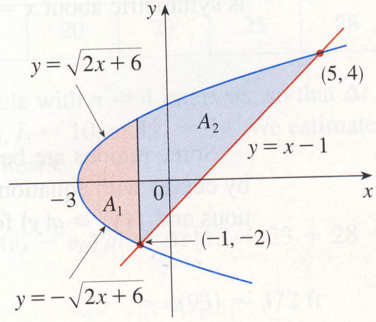
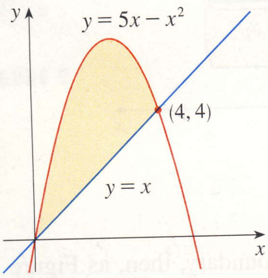


FIGURE 14

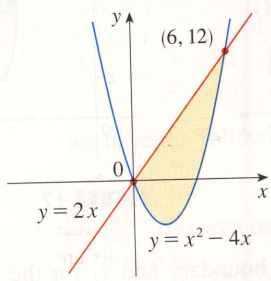
6.1 Exercises

1–4 ||| Find the area of the shaded region.

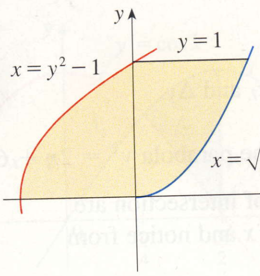
1.



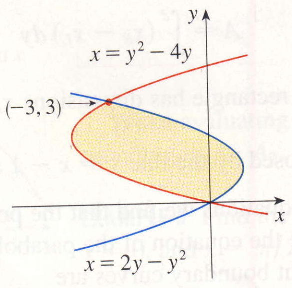
2.



3.



4.



5–26 ||| Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

5. $y = x + 1, \quad y = 9 - x^2, \quad x = -1, \quad x = 2$

6. $y = \sin x, \quad y = x, \quad x = \pi/2, \quad x = \pi$

7. $y = x, \quad y = x^2$

8. $y = x^2, \quad y = x^4$

9. $y = \sqrt{x + 3}, \quad y = (x + 3)/2$

10. $y = 1 + \sqrt{x}, \quad y = (3 + x)/3$

11. $y = x^2, \quad y^2 = x$

12. $y = x, \quad y = \sqrt[3]{x}$

13. $y = 12 - x^2, \quad y = x^2 - 6$

14. $y = x^3 - x, \quad y = 3x$

15. $y = \sqrt{x}, \quad y = \frac{1}{2}x, \quad x = 9$

16. $y = 8 - x^2, \quad y = x^2, \quad x = -3, \quad x = 3$

17. $x = 2y^2, \quad x + y = 1$