3. Find the area under the curve $y = 1/x^3$ from $x = 1$ to $x = t$ and evaluate it for $t = 10$, 100, and 1000. Then find the total area under this curve for $x \geqslant 1$.

4. (a) Graph the functions $f(x) = 1/x^{1.1}$ and $g(x) = 1/x^{0.9}$ in the viewing rectangles $[0, 10]$ by $[0, 1]$ and $[0, 100]$ by $[0, 1]$.
(b) Find the areas under the graphs of f and g from $x = 1$ to $x = t$ and evaluate for $t = 10$, 100, 10^4, 10^6, 10^{10}, and 10^{20}.
(c) Find the total area under each curve for $x \geqslant 1$, if it exists.

5–40 |||| Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

5. $\displaystyle\int_1^\infty \frac{1}{(3x+1)^2}\,dx$ **6.** $\displaystyle\int_{-\infty}^0 \frac{1}{2x-5}\,dx$

7. $\displaystyle\int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}}\,dw$ **8.** $\displaystyle\int_0^\infty \frac{x}{(x^2+2)^2}\,dx$

9. $\displaystyle\int_4^\infty e^{-y/2}\,dy$ **10.** $\displaystyle\int_{-\infty}^{-1} e^{-2t}\,dt$

11. $\displaystyle\int_{-\infty}^\infty \frac{x}{1+x^2}\,dx$ **12.** $\displaystyle\int_{-\infty}^\infty (2 - v^4)\,dv$

13. $\displaystyle\int_{-\infty}^\infty xe^{-x^2}\,dx$ **14.** $\displaystyle\int_{-\infty}^\infty x^2e^{-x^3}\,dx$

15. $\displaystyle\int_{2\pi}^\infty \sin\theta\,d\theta$ **16.** $\displaystyle\int_0^\infty \cos^2\alpha\,d\alpha$

17. $\displaystyle\int_1^\infty \frac{x+1}{x^2+2x}\,dx$ **18.** $\displaystyle\int_0^\infty \frac{dz}{z^2+3z+2}$

19. $\displaystyle\int_0^\infty se^{-5s}\,ds$ **20.** $\displaystyle\int_{-\infty}^6 re^{r/3}\,dr$

21. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx$ **22.** $\displaystyle\int_{-\infty}^\infty e^{-|x|}\,dx$

23. $\displaystyle\int_{-\infty}^\infty \frac{x^2}{9+x^6}\,dx$ **24.** $\displaystyle\int_1^\infty \frac{\ln x}{x^3}\,dx$

25. $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx$ **26.** $\displaystyle\int_0^\infty \frac{x\arctan x}{(1+x^2)^2}\,dx$

27. $\displaystyle\int_0^3 \frac{1}{\sqrt{x}}\,dx$ **28.** $\displaystyle\int_0^3 \frac{1}{x\sqrt{x}}\,dx$

29. $\displaystyle\int_{-1}^0 \frac{1}{x^2}\,dx$ **30.** $\displaystyle\int_1^9 \frac{1}{\sqrt[3]{x-9}}\,dx$

31. $\displaystyle\int_{-2}^3 \frac{1}{x^4}\,dx$ **32.** $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

33. $\displaystyle\int_0^{33} (x-1)^{-1/5}\,dx$ **34.** $\displaystyle\int_0^1 \frac{1}{4y-1}\,dy$

35. $\displaystyle\int_0^\pi \sec x\,dx$ **36.** $\displaystyle\int_0^4 \frac{1}{x^2+x-6}\,dx$

37. $\displaystyle\int_{-1}^1 \frac{e^x}{e^x-1}\,dx$ **38.** $\displaystyle\int_0^2 \frac{x-3}{2x-3}\,dx$

39. $\displaystyle\int_0^2 z^2\ln z\,dz$ **40.** $\displaystyle\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$

41–46 |||| Sketch the region and find its area (if the area is finite).

41. $S = \{(x, y) \mid x \leqslant 1,\ 0 \leqslant y \leqslant e^x\}$

42. $S = \{(x, y) \mid x \geqslant -2,\ 0 \leqslant y \leqslant e^{-x/2}\}$

43. $S = \{(x, y) \mid 0 \leqslant y \leqslant 2/(x^2+9)\}$

44. $S = \{(x, y) \mid x \geqslant 0,\ 0 \leqslant y \leqslant x/(x^2+9)\}$

45. $S = \{(x, y) \mid 0 \leqslant x < \pi/2,\ 0 \leqslant y \leqslant \sec^2 x\}$

46. $S = \{(x, y) \mid -2 < x \leqslant 0,\ 0 \leqslant y \leqslant 1/\sqrt{x+2}\}$

47. (a) If $g(x) = (\sin^2 x)/x^2$, use your calculator or computer t
make a table of approximate values of $\int_1^t g(x)\,dx$ for t =
5, 10, 100, 1000, and 10,000. Does it appear that $\int_1^\infty g($
is convergent?
(b) Use the Comparison Theorem with $f(x) = 1/x^2$ to sh
that $\int_1^\infty g(x)\,dx$ is convergent.
(c) Illustrate part (b) by graphing f and g on the same sc
for $1 \leqslant x \leqslant 10$. Use your graph to explain intuitively
$\int_1^\infty g(x)\,dx$ is convergent.

48. (a) If $g(x) = 1/(\sqrt{x} - 1)$, use your calculator or comput
make a table of approximate values of $\int_2^t g(x)\,dx$ for t
10, 100, 1000, and 10,000. Does it appear that $\int_2^\infty g(x$
convergent or divergent?
(b) Use the Comparison Theorem with $f(x) = 1/\sqrt{x}$ to
that $\int_2^\infty g(x)\,dx$ is divergent.
(c) Illustrate part (b) by graphing f and g on the same s
for $2 \leqslant x \leqslant 20$. Use your graph to explain intuitivel
$\int_2^\infty g(x)\,dx$ is divergent.

49–54 |||| Use the Comparison Theorem to determine wheth
integral is convergent or divergent.

49. $\displaystyle\int_1^\infty \frac{\cos^2 x}{1+x^2}\,dx$ **50.** $\displaystyle\int_1^\infty \frac{2+e^{-x}}{x}\,dx$

51. $\displaystyle\int_1^\infty \frac{dx}{x+e^{2x}}$ **52.** $\displaystyle\int_1^\infty \frac{x}{\sqrt{1+x^6}}\,dx$

53. $\displaystyle\int_0^{\pi/2} \frac{dx}{x\sin x}$ **54.** $\displaystyle\int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx$

55. The integral

$$\int_0^\infty \frac{1}{\sqrt{x}\,(1+x)}\,dx$$

is improper for two reasons: The interval $[0, \infty)$ is infi
the integrand has an infinite discontinuity at 0. Evalua
expressing it as a sum of improper integrals of Type 2
Type 1 as follows:

$$\int_0^\infty \frac{1}{\sqrt{x}\,(1+x)}\,dx = \int_0^1 \frac{1}{\sqrt{x}\,(1+x)}\,dx + \int_1^\infty \frac{1}{\sqrt{x}\,(1$$