

### 8.4 Exercises

**1–6** Write out the form of the partial fraction decomposition of the function (as in Example 7). Do not determine the numerical values of the coefficients.

1. (a)  $\frac{2x}{(x+3)(3x+1)}$

(b)  $\frac{1}{x^3 + 2x^2 + x}$

2. (a)  $\frac{x-1}{x^3 + x^2}$

(b)  $\frac{x-1}{x^3 + x}$

3. (a)  $\frac{2}{x^2 + 3x - 4}$

(b)  $\frac{x^2}{(x-1)(x^2 + x + 1)}$

4. (a)  $\frac{x^3}{x^2 + 4x + 3}$

(b)  $\frac{2x+1}{(x+1)^3(x^2+4)^2}$

5. (a)  $\frac{x^4}{x^4 - 1}$

(b)  $\frac{t^4 + t^2 + 1}{(t^2 + 1)(t^2 + 4)^2}$

6. (a)  $\frac{x^4}{(x^3 + x)(x^2 - x + 3)}$

(b)  $\frac{1}{x^6 - x^3}$

**7–38** Evaluate the integral.

7.  $\int \frac{x}{x-6} dx$

8.  $\int \frac{r^2}{r+4} dr$

9.  $\int \frac{x-9}{(x+5)(x-2)} dx$

10.  $\int \frac{1}{(t+4)(t-1)} dt$

11.  $\int_2^3 \frac{1}{x^2-1} dx$

12.  $\int_0^1 \frac{x-1}{x^2+3x+2} dx$

13.  $\int \frac{ax}{x^2-bx} dx$

14.  $\int \frac{1}{(x+a)(x+b)} dx$

15.  $\int_0^1 \frac{2x+3}{(x+1)^2} dx$

16.  $\int_0^1 \frac{x^3-4x-10}{x^2-x-6} dx$

17.  $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)} dy$

18.  $\int \frac{x^2+2x-1}{x^3-x} dx$

19.  $\int \frac{1}{(x+5)^2(x-1)} dx$

20.  $\int \frac{x^2}{(x-3)(x+2)^2} dx$

21.  $\int \frac{5x^2+3x-2}{x^3+2x^2} dx$

22.  $\int \frac{ds}{s^2(s-1)^2}$

23.  $\int \frac{x^2}{(x+1)^3} dx$

24.  $\int \frac{x^3}{(x+1)^3} dx$

25.  $\int \frac{10}{(x-1)(x^2+9)} dx$

26.  $\int \frac{x^2-x+6}{x^3+3x} dx$

27.  $\int \frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)} dx$

28.  $\int \frac{x^2-2x-1}{(x-1)^2(x^2+1)} dx$

29.  $\int \frac{x+4}{x^2+2x+5} dx$

30.  $\int \frac{x^3-2x^2+x+1}{x^4+5x^2+4} dx$

31.  $\int \frac{1}{x^3-1} dx$

32.  $\int_0^1 \frac{x}{x^2+4x+13} dx$

33.  $\int_2^5 \frac{x^2+2x}{x^3+3x^2+4} dx$

34.  $\int \frac{x^3}{x^3+1} dx$

35.  $\int \frac{dx}{x^4-x^2}$

36.  $\int_0^1 \frac{2x^3+5x}{x^4+5x^2+4} dx$

37.  $\int \frac{x-3}{(x^2+2x+4)^2} dx$

38.  $\int \frac{x^4+1}{x(x^2+1)^2} dx$

**39–48** Make a substitution to express the integrand as a rational function and then evaluate the integral.

39.  $\int \frac{1}{x\sqrt{x+1}} dx$

40.  $\int \frac{1}{x-\sqrt{x+2}} dx$

41.  $\int_9^{16} \frac{\sqrt{x}}{x-4} dx$

42.  $\int_0^1 \frac{1}{1+\sqrt[3]{x}} dx$

43.  $\int \frac{x^3}{\sqrt[3]{x^2+1}} dx$

44.  $\int_{1/3}^3 \frac{\sqrt{x}}{x^2+x} dx$

45.  $\int \frac{1}{\sqrt{x}-\sqrt[3]{x}} dx$  [Hint: Substitute  $u = \sqrt[6]{x}$ .]

46.  $\int \frac{1}{\sqrt[3]{x}+\sqrt[4]{x}} dx$  [Hint: Substitute  $u = \sqrt[12]{x}$ .]

47.  $\int \frac{e^{2x}}{e^{2x}+3e^x+2} dx$

48.  $\int \frac{\cos x}{\sin^2 x + \sin x} dx$

**49–50** Use integration by parts, together with the techniques of this section, to evaluate the integral.

49.  $\int \ln(x^2-x+2) dx$

50.  $\int x \tan^{-1} x dx$

**51.** Use a graph of  $f(x) = 1/(x^2 - 2x - 3)$  to decide whether  $\int_0^2 f(x) dx$  is positive or negative. Use the graph to give a rough estimate of the value of the integral and then use partial fractions to find the exact value.

**52.** Graph both  $y = 1/(x^3 - 2x^2)$  and an antiderivative on the same screen.

**53–54** Evaluate the integral by completing the square and using Formula 6.

53.  $\int \frac{dx}{x^2-2x}$

54.  $\int \frac{2x+1}{4x^2+12x-7} dx$

**55.** The German mathematician Karl Weierstrass (1815–1897) noticed that the substitution  $t = \tan(x/2)$  will convert any