

11.  $\int \sqrt{1-4x^2} dx$

13.  $\int \frac{\sqrt{x^2-9}}{x^3} dx$

15.  $\int \frac{x^2}{(a^2-x^2)^{3/2}} dx$

17.  $\int \frac{x}{\sqrt{x^2-7}} dx$

19.  $\int \frac{\sqrt{1+x^2}}{x} dx$

21.  $\int_0^{2/3} x^3 \sqrt{4-9x^2} dx$

23.  $\int \sqrt{5+4x-x^2} dx$

25.  $\int \frac{1}{\sqrt{9x^2+6x-8}} dx$

27.  $\int \frac{dx}{(x^2+2x+2)^2}$

29.  $\int x\sqrt{1-x^4} dx$

12.  $\int_0^1 x\sqrt{x^2+4} dx$

14.  $\int \frac{du}{u\sqrt{5-u^2}}$

16.  $\int \frac{dx}{x^2\sqrt{16x^2-9}}$

18.  $\int \frac{dx}{[(ax)^2-b^2]^{3/2}}$

20.  $\int \frac{t}{\sqrt{25-t^2}} dt$

22.  $\int_0^1 \sqrt{x^2+1} dx$

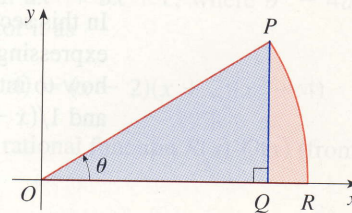
24.  $\int \frac{dt}{\sqrt{t^2-6t+13}}$

26.  $\int \frac{x^2}{\sqrt{4x-x^2}} dx$

28.  $\int \frac{dx}{(5-4x-x^2)^{5/2}}$

30.  $\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}} dt$

equation  $x^2 + y^2 = r^2$ . Then  $A$  is the sum of the area of the triangle  $POQ$  and the area of the region  $PQR$  in the figure.]



36. Evaluate the integral

$$\int \frac{dx}{x^4 \sqrt{x^2-2}}$$

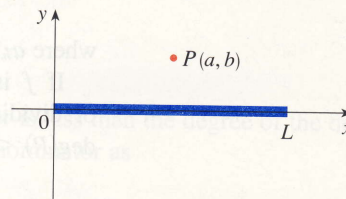
Graph the integrand and its indefinite integral on the same screen and check that your answer is reasonable.

37. Use a graph to approximate the roots of the equation  $x^2\sqrt{4-x^2} = 2-x$ . Then approximate the area bounded by the curve  $y = x^2\sqrt{4-x^2}$  and the line  $y = 2-x$ .

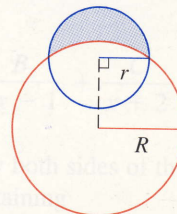
38. A charged rod of length  $L$  produces an electric field at point  $P(a, b)$  given by

$$E(P) = \int_{-a}^{L-a} \frac{\lambda b}{4\pi\epsilon_0(x^2+b^2)^{3/2}} dx$$

where  $\lambda$  is the charge density per unit length on the rod and  $\epsilon_0$  is the free space permittivity (see the figure). Evaluate the integral to determine an expression for the electric field  $E(P)$ .



39. Find the area of the crescent-shaped region (called a *lune*) bounded by arcs of circles with radii  $r$  and  $R$ . (See the figure.)



31. (a) Use trigonometric substitution to show that

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2}) + C$$

(b) Use the hyperbolic substitution  $x = a \sinh t$  to show that

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

These formulas are connected by Formula 7.6.3.

32. Evaluate

$$\int \frac{x^2}{(x^2+a^2)^{3/2}} dx$$

(a) by trigonometric substitution.

(b) by the hyperbolic substitution  $x = a \sinh t$ .

33. Find the average value of  $f(x) = \sqrt{x^2-1}/x$ ,  $1 \leq x \leq 7$ .

34. Find the area of the region bounded by the hyperbola  $9x^2 - 4y^2 = 36$  and the line  $x = 3$ .

35. Prove the formula  $A = \frac{1}{2}r^2\theta$  for the area of a sector of a circle with radius  $r$  and central angle  $\theta$ . [Hint: Assume  $0 < \theta < \pi/2$  and place the center of the circle at the origin so it has the

40. A water storage tank has the shape of a cylinder with diameter 10 ft. It is mounted so that the circular cross-sections are vertical. If the depth of the water is 7 ft, what percentage of the total capacity is being used?

41. A torus is generated by rotating the circle  $x^2 + (y-R)^2 = r^2$  about the  $x$ -axis. Find the volume enclosed by the torus.