EXAMPLE 9 Evaluate $\int \sin 4x \cos 5x \, dx$.

SOLUTION This integral could be evaluated using integration by parts, but it's easier to use the identity in Equation 2(a) as follows:

$$\int \sin 4x \cos 5x \, dx = \int \tfrac{1}{2}[\sin(-x) + \sin 9x] \, dx$$

$$= \tfrac{1}{2} \int (-\sin x + \sin 9x) \, dx$$

$$= \tfrac{1}{2}\left(\cos x - \tfrac{1}{9}\cos 9x\right) + C$$

▍▍▍ **8.2** Exercises

1–47 ▍▍▍ Evaluate the integral.

1. $\int \sin^3 x \, \cos^2 x \, dx$

2. $\int \sin^6 x \, \cos^3 x \, dx$

3. $\int_{\pi/2}^{3\pi/4} \sin^5 x \, \cos^3 x \, dx$

4. $\int_0^{\pi/2} \cos^5 x \, dx$

5. $\int \cos^5 x \, \sin^4 x \, dx$

6. $\int \sin^3(mx) \, dx$

7. $\int_0^{\pi/2} \cos^2 \theta \, d\theta$

8. $\int_0^{\pi/2} \sin^2(2\theta) \, d\theta$

9. $\int_0^{\pi} \sin^4(3t) \, dt$

10. $\int_0^{\pi} \cos^6 \theta \, d\theta$

11. $\int (1 + \cos \theta)^2 \, d\theta$

12. $\int x \cos^2 x \, dx$

13. $\int_0^{\pi/4} \sin^4 x \, \cos^2 x \, dx$

14. $\int_0^{\pi/2} \sin^2 x \, \cos^2 x \, dx$

15. $\int \sin^3 x \, \sqrt{\cos x} \, dx$

16. $\int \cos \theta \, \cos^5(\sin \theta) \, d\theta$

17. $\int \cos^2 x \, \tan^3 x \, dx$

18. $\int \cot^5 \theta \, \sin^4 \theta \, d\theta$

19. $\int \dfrac{1 - \sin x}{\cos x} \, dx$

20. $\int \cos^2 x \, \sin 2x \, dx$

21. $\int \sec^2 x \, \tan x \, dx$

22. $\int_0^{\pi/2} \sec^4(t/2) \, dt$

23. $\int \tan^2 x \, dx$

24. $\int \tan^4 x \, dx$

25. $\int \sec^6 t \, dt$

26. $\int_0^{\pi/4} \sec^4 \theta \, \tan^4 \theta \, d\theta$

27. $\int_0^{\pi/3} \tan^5 x \, \sec^4 x \, dx$

28. $\int \tan^3(2x) \, \sec^5(2x) \, dx$

29. $\int \tan^3 x \, \sec x \, dx$

30. $\int_0^{\pi/3} \tan^5 x \, \sec^6 x \, dx$

31. $\int \tan^5 x \, dx$

32. $\int \tan^6(ay) \, dy$

33. $\int \dfrac{\tan^3 \theta}{\cos^4 \theta} \, d\theta$

34. $\int \tan^2 x \, \sec x \, dx$

35. $\int_{\pi/6}^{\pi/2} \cot^2 x \, dx$

36. $\int_{\pi/4}^{\pi/2} \cot^3 x \, dx$

37. $\int \cot^3 \alpha \, \csc^3 \alpha \, d\alpha$

38. $\int \csc^4 x \, \cot^6 x \, dx$

39. $\int \csc x \, dx$

40. $\int_{\pi/6}^{\pi/3} \csc^3 x \, dx$

41. $\int \sin 5x \, \sin 2x \, dx$

42. $\int \sin 3x \, \cos x \, dx$

43. $\int \cos 7\theta \, \cos 5\theta \, d\theta$

44. $\int \dfrac{\cos x + \sin x}{\sin 2x} \, dx$

45. $\int \dfrac{1 - \tan^2 x}{\sec^2 x} \, dx$

46. $\int \dfrac{dx}{\cos x - 1}$

47. $\int t \sec^2(t^2) \, \tan^4(t^2) \, dt$

48. If $\int_0^{\pi/4} \tan^6 x \, \sec x \, dx = I$, express the value of $\int_0^{\pi/4} \tan^8 x \, \sec x \, dx$ in terms of I.

49–52 ▍▍▍ Evaluate the indefinite integral. Illustrate, and check that your answer is reasonable, by graphing both the integrand and its antiderivative (taking $C = 0$).

49. $\int \sin^5 x \, dx$

50. $\int \sin^4 x \, \cos^4 x \, dx$

51. $\int \sin 3x \, \sin 6x \, dx$

52. $\int \sec^4 \dfrac{x}{2} \, dx$

53. Find the average value of the function $f(x) = \sin^2 x \, \cos^3 x$ on the interval $[-\pi, \pi]$.