7–32 ||| Evaluate the indefinite integral.

7. $\int 2x(x^2 + 3)^4\, dx$

8. $\int x^2(x^3 + 5)^9\, dx$

9. $\int (3x - 2)^{20}\, dx$

10. $\int (2 - x)^6\, dx$

11. $\int \frac{1 + 4x}{\sqrt{1 + x + 2x^2}}\, dx$

12. $\int \frac{x}{(x^2 + 1)^2}\, dx$

13. $\int \frac{3}{(2y + 1)^5}\, dy$

14. $\int \frac{1}{(5t + 4)^{2.7}}\, dt$

15. $\int \sqrt{4 - t}\, dt$

16. $\int y^3\sqrt{2y^4 - 1}\, dy$

17. $\int \sin \pi t\, dt$

18. $\int \sec 2\theta \tan 2\theta\, d\theta$

19. $\int \frac{\cos\sqrt{t}}{\sqrt{t}}\, dt$

20. $\int \sqrt{x}\sin(1 + x^{3/2})\, dx$

21. $\int \cos\theta \sin^6\theta\, d\theta$

22. $\int (1 + \tan\theta)^5 \sec^2\theta\, d\theta$

23. $\int \frac{z^2}{\sqrt[3]{1 + z^3}}\, dz$

24. $\int \frac{ax + b}{\sqrt{ax^2 + 2bx + c}}\, dx$

25. $\int \sqrt{\cot x}\csc^2 x\, dx$

26. $\int \frac{\cos(\pi/x)}{x^2}\, dx$

27. $\int \sec^3 x \tan x\, dx$

28. $\int \sqrt[3]{x^3 + 1}\, x^5\, dx$

29. $\int x^a\sqrt{b + cx^{a+1}}\, dx \quad (c \neq 0, a \neq -1)$

30. $\int \sin t \sec^2(\cos t)\, dt$

31. $\int \frac{x}{\sqrt[4]{x + 2}}\, dx$

32. $\int \frac{x^2}{\sqrt{1 - x}}\, dx$

33–36 ||| Evaluate the indefinite integral. Illustrate and check that your answer is reasonable by graphing both the function and its antiderivative (take $C = 0$).

33. $\int \frac{3x - 1}{(3x^2 - 2x + 1)^4}\, dx$

34. $\int \frac{x}{\sqrt{x^2 + 1}}\, dx$

35. $\int \sin^3 x \cos x\, dx$

36. $\int \tan^2\theta \sec^2\theta\, d\theta$

37–54 ||| Evaluate the definite integral, if it exists.

37. $\int_0^2 (x - 1)^{25}\, dx$

38. $\int_0^7 \sqrt{4 + 3x}\, dx$

39. $\int_0^1 x^2(1 + 2x^3)^5\, dx$

40. $\int_0^{\sqrt{\pi}} x\cos(x^2)\, dx$

41. $\int_0^{\pi} \sec^2(t/4)\, dt$

42. $\int_{1/6}^{1/2} \csc \pi t \cot \pi t\, dt$

43. $\int_{-\pi/6}^{\pi/6} \tan^3\theta\, d\theta$

44. $\int_0^2 \frac{dx}{(2x - 3)^2}$

45. $\int_0^{\pi/3} \frac{\sin\theta}{\cos^2\theta}\, d\theta$

46. $\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1 + x^6}\, dx$

47. $\int_0^{13} \frac{dx}{\sqrt[3]{(1 + 2x)^2}}$

48. $\int_0^{\pi/2} \cos x \sin(\sin x)\, dx$

49. $\int_1^2 x\sqrt{x - 1}\, dx$

50. $\int_0^4 \frac{x}{\sqrt{1 + 2x}}\, dx$

51. $\int_0^4 \frac{dx}{(x - 2)^3}$

52. $\int_0^a x\sqrt{a^2 - x^2}\, dx$

53. $\int_0^a x\sqrt{x^2 + a^2}\, dx \quad (a > 0)$

54. $\int_{-a}^{a} x\sqrt{x^2 + a^2}\, dx$

55–56 ||| Use a graph to give a rough estimate of the area of the region that lies under the given curve. Then find the exact area.

55. $y = \sqrt{2x + 1}, \quad 0 \leqslant x \leqslant 1$

56. $y = 2\sin x - \sin 2x, \quad 0 \leqslant x \leqslant \pi$

57. Evaluate $\int_{-2}^{2} (x + 3)\sqrt{4 - x^2}\, dx$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

58. Evaluate $\int_0^1 x\sqrt{1 - x^4}\, dx$ by making a substitution and interpreting the resulting integral in terms of an area.

59. Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 s. The maximum rate of air flow into the lungs is about 0.5 L/s. This explains, in part, why the function $f(t) = \frac{1}{2}\sin(2\pi t/5)$ has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time t.

60. Alabama Instruments Company has set up a production line to manufacture a new calculator. The rate of production of these calculators after t weeks is

$$\frac{dx}{dt} = 5000\left(1 - \frac{100}{(t + 10)^2}\right) \text{ calculators/week}$$

(Notice that production approaches 5000 per week as time goes on, but the initial production is lower because of the workers' unfamiliarity with the new techniques.) Find the number of calculators produced from the beginning of the third week to the end of the fourth week.

61. If f is continuous and $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$.

62. If f is continuous and $\int_0^9 f(x)\,dx = 4$, find $\int_0^3 x f(x^2)\,dx$.

63. Suppose f is continuous on $\mathbb{R}$.
(a) Prove that

$$\int_a^b f(-x)\,dx = \int_{-b}^{-a} f(x)\,dx$$

For the case where $f(x) \geqslant 0$ and $0 < a < b$, draw a diagram to interpret this equation geometrically as an equality of areas.
(b) Prove that

$$\int_a^b f(x+c)\,dx = \int_{a+c}^{b+c} f(x)\,dx$$

For the case where $f(x) \geqslant 0$, draw a diagram to interpret this equation geometrically as an equality of areas.

64. Show that the area under the graph of $y = \sin\sqrt{x}$ from 0 to 4 is the same as the area under the graph of $y = 2x \sin x$ from 0 to 2.

65. If a and b are positive numbers, show that

$$\int_0^1 x^a(1-x)^b\,dx = \int_0^1 x^b(1-x)^a\,dx$$

66. Use the substitution $u = \pi - x$ to show that

$$\int_0^\pi x f(\sin x)\,dx = \frac{\pi}{2}\int_0^\pi f(\sin x)\,dx$$

The following exercises are intended only for those who have already covered Chapter 7.

67–82 Evaluate the integral.

67. $\displaystyle\int \frac{dx}{5-3x}$

68. $\displaystyle\int \frac{x}{x^2+1}\,dx$

69. $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

70. $\displaystyle\int \frac{\tan^{-1}x}{1+x^2}\,dx$

71. $\displaystyle\int e^x\sqrt{1+e^x}\,dx$

72. $\displaystyle\int e^{\cos t}\sin t\,dt$

73. $\displaystyle\int \frac{dx}{x\ln x}$

74. $\displaystyle\int \frac{e^x}{e^x+1}\,dx$

75. $\displaystyle\int \cot x\,dx$

76. $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

77. $\displaystyle\int \frac{1+x}{1+x^2}\,dx$

78. $\displaystyle\int \frac{x}{1+x^4}\,dx$

79. $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,dx$

80. $\displaystyle\int_0^1 xe^{-x^2}\,dx$

81. $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

82. $\displaystyle\int_0^{1/2} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

83. Use Exercise 66 to evaluate the integral

$$\int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$$