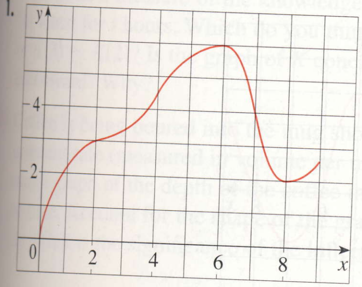
4.3 Exercises

1–2 Use the given graph of f to find the following.

(a) The largest open intervals on which f is increasing.
(b) The largest open intervals on which f is decreasing.
(c) The largest open intervals on which f is concave upward.
(d) The largest open intervals on which f is concave downward.
(e) The coordinates of the points of inflection.

1.



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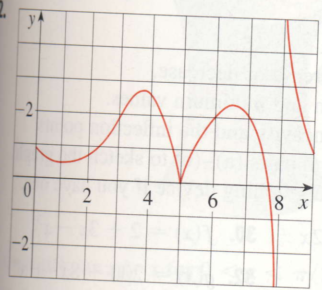
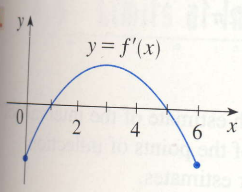


3. Suppose you are given a formula for a function f.
(a) How do you determine where f is increasing or decreasing?
(b) How do you determine where the graph of f is concave upward or concave downward?
(c) How do you locate inflection points?

4. (a) State the First Derivative Test.
(b) State the Second Derivative Test. Under what circumstances is it inconclusive? What do you do if it fails?

5–6 The graph of the *derivative* f' of a function f is shown.
(a) On what intervals is f increasing or decreasing?
(b) At what values of x does f have a local maximum or minimum?

5.



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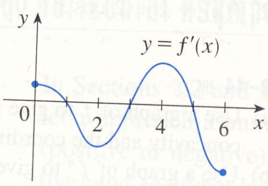
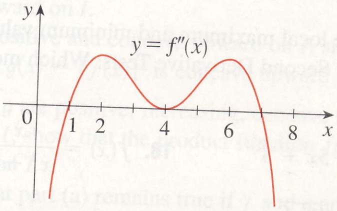
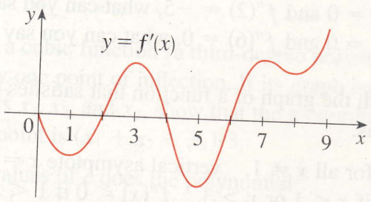


7. The graph of the second derivative f'' of a function f is shown. State the x-coordinates of the inflection points of f. Give reasons for your answers.

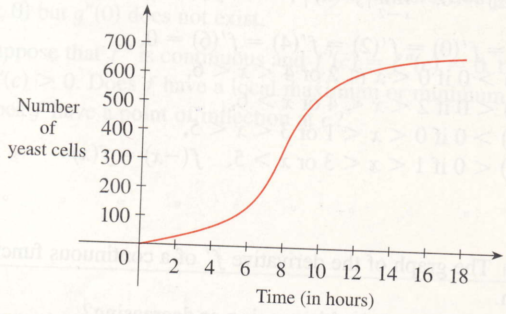


8. The graph of the first derivative f' of a function f is shown.
(a) On what intervals is f increasing? Explain.
(b) At what values of x does f have a local maximum or minimum? Explain.
(c) On what intervals is f concave upward or concave downward? Explain.
(d) What are the x-coordinates of the inflection points of f? Why?



9. Sketch the graph of a function whose first and second derivatives are always negative.

10. A graph of a population of yeast cells in a new laboratory culture as a function of time is shown.
(a) Describe how the rate of population increase varies.
(b) When is this rate highest?
(c) On what intervals is the population function concave upward or downward?
(d) Estimate the coordinates of the inflection point.



11–16
(a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.

11. $f(x) = x^3 - 12x + 1$

12. $f(x) = 5 - 3x^2 + x^3$

13. $f(x) = x^4 - 2x^2 + 3$ **14.** $f(x) = \dfrac{x^2}{x^2 + 3}$

15. $f(x) = x - 2 \sin x, \quad 0 < x < 3\pi$

16. $f(x) = \cos^2 x - 2 \sin x, \quad 0 \leqslant x \leqslant 2\pi$

17–19 ||| Find the local maximum and minimum values of f using both the First and Second Derivative Tests. Which method do you prefer?

17. $f(x) = x^5 - 5x + 3$ **18.** $f(x) = \dfrac{x}{x^2 + 4}$

19. $f(x) = x + \sqrt{1 - x}$

20. (a) Find the critical numbers of $f(x) = x^4(x - 1)^3$.
(b) What does the Second Derivative Test tell you about the behavior of f at these critical numbers?
(c) What does the First Derivative Test tell you?

21. Suppose f'' is continuous on $(-\infty, \infty)$.
(a) If $f'(2) = 0$ and $f''(2) = -5$, what can you say about f?
(b) If $f'(6) = 0$ and $f''(6) = 0$, what can you say about f?

22–26 ||| Sketch the graph of a function that satisfies all of the given conditions.

22. $f'(x) > 0$ for all $x \neq 1$, vertical asymptote $x = 1$,
$f''(x) > 0$ if $x < 1$ or $x > 3$, $f''(x) < 0$ if $1 < x < 3$

23. $f'(0) = f'(2) = f'(4) = 0$,
$f'(x) > 0$ if $x < 0$ or $2 < x < 4$,
$f'(x) < 0$ if $0 < x < 2$ or $x > 4$,
$f''(x) > 0$ if $1 < x < 3$, $f''(x) < 0$ if $x < 1$ or $x > 3$

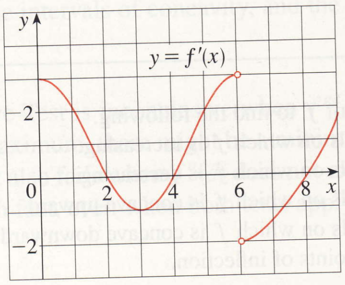
24. $f'(1) = f'(-1) = 0$, $f'(x) < 0$ if $|x| < 1$,
$f'(x) > 0$ if $1 < |x| < 2$, $f'(x) = -1$ if $|x| > 2$,
$f''(x) < 0$ if $-2 < x < 0$, inflection point $(0, 1)$

25. $f'(x) > 0$ if $|x| < 2$, $f'(x) < 0$ if $|x| > 2$,
$f'(-2) = 0$, $\lim_{x \to 2} |f'(x)| = \infty$, $f''(x) > 0$ if $x \neq 2$

26. $f(0) = f'(0) = f'(2) = f'(4) = f'(6) = 0$,
$f'(x) > 0$ if $0 < x < 2$ or $4 < x < 6$,
$f'(x) < 0$ if $2 < x < 4$ or $x > 6$,
$f''(x) > 0$ if $0 < x < 1$ or $3 < x < 5$,
$f''(x) < 0$ if $1 < x < 3$ or $x > 5$, $f(-x) = f(x)$

27–28 ||| The graph of the derivative f' of a continuous function f is shown.
(a) On what intervals is f increasing or decreasing?
(b) At what values of x does f have a local maximum or minimum?
(c) On what intervals is f concave upward or downward?
(d) State the x-coordinate(s) of the point(s) of inflection.
(e) Assuming that $f(0) = 0$, sketch a graph of f.

27.



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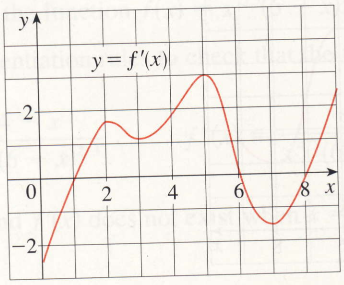

29–40 |||
(a) Find the intervals of increase or decrease.
(b) Find the local maximum and minimum values.
(c) Find the intervals of concavity and the inflection points.
(d) Use the information from parts (a)–(c) to sketch the gra
Check your work with a graphing device if you have or

29. $f(x) = 2x^3 - 3x^2 - 12x$ **30.** $f(x) = 2 + 3x - x^3$

31. $f(x) = x^4 - 6x^2$ **32.** $g(x) = 200 + 8x^3 +$

33. $h(x) = 3x^5 - 5x^3 + 3$ **34.** $h(x) = (x^2 - 1)^3$

35. $A(x) = x\sqrt{x + 3}$ **36.** $G(x) = x - 4\sqrt{x}$

37. $C(x) = x^{1/3}(x + 4)$ **38.** $B(x) = 3x^{2/3} - x$

39. $f(\theta) = 2 \cos \theta - \cos 2\theta, \quad 0 \leqslant \theta \leqslant 2\pi$

40. $f(t) = t + \cos t, \quad -2\pi \leqslant t \leqslant 2\pi$

41–42 |||
(a) Use a graph of f to estimate the maximum and minim values. Then find the exact values.
(b) Estimate the value of x at which f increases most rapi find the exact value.

41. $f(x) = \dfrac{x + 1}{\sqrt{x^2 + 1}}$

42. $f(x) = x + 2 \cos x, \quad 0 \leqslant x \leqslant 2\pi$

43–44 |||
(a) Use a graph of f to give a rough estimate of the inter concavity and the coordinates of the points of inflecti
(b) Use a graph of f'' to give better estimates.

43. $f(x) = \cos x + \frac{1}{2} \cos 2x, \quad 0 \leqslant x \leqslant 2\pi$