31–44 Find the critical numbers of the function.

31. $f(x) = 5x^2 + 4x$ **32.** $f(x) = x^3 + x^2 - x$

33. $f(x) = x^3 + 3x^2 - 24x$ **34.** $f(x) = x^3 + x^2 + x$

35. $s(t) = 3t^4 + 4t^3 - 6t^2$ **36.** $f(z) = \dfrac{z+1}{z^2 + z + 1}$

37. $g(x) = |2x + 3|$ **38.** $g(x) = x^{1/3} - x^{-2/3}$

39. $g(t) = 5t^{2/3} + t^{5/3}$ **40.** $g(t) = \sqrt{t}\,(1 - t)$

41. $F(x) = x^{4/5}(x - 4)^2$ **42.** $G(x) = \sqrt[3]{x^2 - x}$

43. $f(\theta) = 2\cos\theta + \sin^2\theta$ **44.** $g(\theta) = 4\theta - \tan\theta$

45–56 Find the absolute maximum and absolute minimum values of f on the given interval.

45. $f(x) = 3x^2 - 12x + 5$, $[0, 3]$

46. $f(x) = x^3 - 3x + 1$, $[0, 3]$

47. $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$

48. $f(x) = x^3 - 6x^2 + 9x + 2$, $[-1, 4]$

49. $f(x) = x^4 - 2x^2 + 3$, $[-2, 3]$

50. $f(x) = (x^2 - 1)^3$, $[-1, 2]$

51. $f(x) = \dfrac{x}{x^2 + 1}$, $[0, 2]$

52. $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$, $[-4, 4]$

53. $f(t) = t\sqrt{4 - t^2}$, $[-1, 2]$

54. $f(t) = \sqrt[3]{t}\,(8 - t)$, $[0, 8]$

55. $f(x) = \sin x + \cos x$, $[0, \pi/3]$

56. $f(x) = x - 2\cos x$, $[-\pi, \pi]$

57. If a and b are positive numbers, find the maximum value of $f(x) = x^a(1 - x)^b$, $0 \leqslant x \leqslant 1$.

58. Use a graph to estimate the critical numbers of $f(x) = |x^3 - 3x^2 + 2|$ correct to one decimal place.

59–62

(a) Use a graph to estimate the absolute maximum and minimum values of the function to two decimal places.

(b) Use calculus to find the exact maximum and minimum values.

59. $f(x) = x^3 - 8x + 1$, $-3 \leqslant x \leqslant 3$

60. $f(x) = x^4 - 3x^3 + 3x^2 - x$, $0 \leqslant x \leqslant 2$

61. $f(x) = x\sqrt{x - x^2}$

62. $f(x) = (\cos x)/(2 + \sin x)$, $0 \leqslant x \leqslant 2\pi$

63. Between 0°C and 30°C, the volume V (in cubic centimeters) of 1 kg of water at a temperature T is given approximately by the formula

$$V = 999.87 - 0.06426T + 0.0085043T^2 - 0.0000679T^3$$

Find the temperature at which water has its maximum density.

64. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

where μ is a positive constant called the *coefficient of friction* and where $0 \leqslant \theta \leqslant \pi/2$. Show that F is minimized when $\tan\theta = \mu$.

65. A model for the food-price index (the price of a representative "basket" of foods) between 1984 and 1994 is given by the function

$$I(t) = 0.00009045t^5 + 0.001438t^4 - 0.06561t^3 + 0.4598t^2 - 0.6270t + 99.33$$

where t is measured in years since midyear 1984, so $0 \leqslant t \leqslant 10$, and $I(t)$ is measured in 1987 dollars and scaled such that $I(3) = 100$. Estimate the times when food was cheapest and most expensive during the period 1984–1994.

66. On May 7, 1992, the space shuttle *Endeavour* was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters.

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

(a) Use a graphing calculator or computer to find the cubic polynomial that best models the velocity of the shuttle for the time interval $t \in [0, 125]$. Then graph this polynomial.

(b) Find a model for the acceleration of the shuttle and use it to estimate the maximum and minimum values of the acceleration during the first 125 seconds.

67. When a foreign object lodged in the trachea (windpipe) forces a person to cough, the diaphragm thrusts upward causing an increase in pressure in the lungs. This is accompanied by a