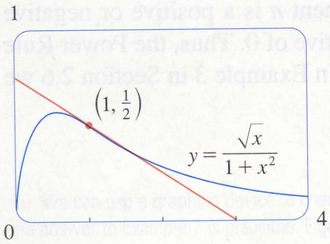

FIGURE 5

So the slope of the tangent line at $(1, \frac{1}{2})$ is

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1 - 3 \cdot 1^2}{2\sqrt{1}(1 + 1^2)^2} = -\frac{1}{4}$$

We use the point-slope form to write an equation of the tangent line at $(1, \frac{1}{2})$:

$$y - \tfrac{1}{2} = -\tfrac{1}{4}(x - 1) \qquad \text{or} \qquad y = -\tfrac{1}{4}x + \tfrac{3}{4}$$

The curve and its tangent line are graphed in Figure 5.

EXAMPLE 11 At what points on the hyperbola $xy = 12$ is the tangent line parallel to the line $3x + y = 0$?

SOLUTION Since $xy = 12$ can be written as $y = 12/x$, we have

$$\frac{dy}{dx} = 12\frac{d}{dx}(x^{-1}) = 12(-x^{-2}) = -\frac{12}{x^2}$$

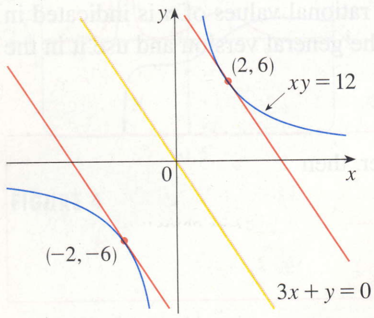

FIGURE 6

Let the x-coordinate of one of the points in question be a. Then the slope of the tangent line at that point is $-12/a^2$. This tangent line will be parallel to the line $3x + y = 0$, or $y = -3x$, if it has the same slope, that is, -3. Equating slopes, we get

$$-\frac{12}{a^2} = -3 \qquad \text{or} \qquad a^2 = 4 \qquad \text{or} \qquad a = \pm 2$$

Therefore, the required points are $(2, 6)$ and $(-2, -6)$. The hyperbola and the tangents are shown in Figure 6.

We summarize the differentiation formulas we have learned so far as follows.

Table of Differentiation Formulas

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$	
$(cf)' = cf'$	$(f + g)' = f' + g'$	$(f - g)' = f' - g'$
$(fg)' = fg' + gf'$	$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$	

3.3 Exercises

1–20 ||| Differentiate the function.

1. $f(x) = 186.5$
2. $f(x) = \sqrt{30}$
3. $f(x) = 5x - 1$
4. $F(x) = -4x^{10}$
5. $f(x) = x^2 + 3x - 4$
6. $g(x) = 5x^8 - 2x^5 + 6$
7. $f(t) = \frac{1}{4}(t^4 + 8)$
8. $f(t) = \frac{1}{2}t^6 - 3t^4 + t$
9. $V(r) = \frac{4}{3}\pi r^3$
10. $R(t) = 5t^{-3/5}$
11. $Y(t) = 6t^{-9}$
12. $R(x) = \frac{\sqrt{10}}{x^7}$
13. $F(x) = (\frac{1}{2}x)^5$
14. $f(t) = \sqrt{t} - \frac{1}{\sqrt{t}}$
15. $y = x^{-2/5}$
16. $y = \sqrt[3]{x}$
17. $y = 4\pi^2$
18. $g(u) = \sqrt{2}u + \sqrt{3u}$

19. $v = t^2 - \dfrac{1}{\sqrt[4]{t^3}}$

20. $u = \sqrt[3]{t^2} + 2\sqrt{t^3}$

21. Find the derivative of $y = (x^2 + 1)(x^3 + 1)$ in two ways: by using the Product Rule and by performing the multiplication first. Do your answers agree?

22. Find the derivative of the function

$$F(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}}$$

in two ways: by using the Quotient Rule and by simplifying first. Show that your answers are equivalent. Which method do you prefer?

23–42 ||| Differentiate.

23. $V(x) = (2x^3 + 3)(x^4 - 2x)$

24. $Y(u) = (u^{-2} + u^{-3})(u^5 - 2u^2)$

25. $F(y) = \left(\dfrac{1}{y^2} - \dfrac{3}{y^4}\right)(y + 5y^3)$

26. $y = \sqrt{x}\,(x - 1)$

27. $g(x) = \dfrac{3x - 1}{2x + 1}$

28. $f(t) = \dfrac{2t}{4 + t^2}$

29. $y = \dfrac{t^2}{3t^2 - 2t + 1}$

30. $y = \dfrac{t^3 + t}{t^4 - 2}$

31. $y = \dfrac{v^3 - 2v\sqrt{v}}{v}$

32. $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$

33. $y = \dfrac{1}{x^4 + x^2 + 1}$

34. $y = x^2 + x + x^{-1} + x^{-2}$

35. $y = ax^2 + bx + c$

36. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2}$

37. $y = \dfrac{r^2}{1 + \sqrt{r}}$

38. $y = \dfrac{cx}{1 + cx}$

39. $y = \sqrt[5]{t}\,(t^2 + t + t^{-1})$

40. $y = \dfrac{u^6 - 2u^3 + 5}{u^2}$

41. $f(x) = \dfrac{x}{x + \dfrac{c}{x}}$

42. $f(x) = \dfrac{ax + b}{cx + d}$

43. The general polynomial of degree n has the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where $a_n \neq 0$. Find the derivative of P.

44–46 ||| Find $f'(x)$. Compare the graphs of f and f' and use them to explain why your answer is reasonable.

44. $f(x) = x/(x^2 - 1)$

45. $f(x) = 3x^{15} - 5x^3 + 3$

46. $f(x) = x + \dfrac{1}{x}$

47–48 ||| Estimate the value of $f'(a)$ by zooming in on the graph of f. Then differentiate f to find the exact value of $f'(a)$ and compare with your estimate.

47. $f(x) = 3x^2 - x^3, \quad a = 1$

48. $f(x) = 1/\sqrt{x}, \quad a = 4$

49. (a) Use a graphing calculator or computer to graph the function $f(x) = x^4 - 3x^3 - 6x^2 + 7x + 30$ in the viewing rectangle $[-3, 5]$ by $[-10, 50]$.
(b) Using the graph in part (a) to estimate slopes, make a rough sketch, by hand, of the graph of f'. (See Example 1 in Section 3.2.)
(c) Calculate $f'(x)$ and use this expression, with a graphing device, to graph f'. Compare with your sketch in part (b).

50. (a) Use a graphing calculator or computer to graph the function $g(x) = x^2/(x^2 + 1)$ in the viewing rectangle $[-4, 4]$ by $[-1, 1.5]$.
(b) Using the graph in part (a) to estimate slopes, make a rough sketch, by hand, of the graph of g'. (See Example 1 in Section 3.2.)
(c) Calculate $g'(x)$ and use this expression, with a graphing device, to graph g'. Compare with your sketch in part (b).

51–54 ||| Find an equation of the tangent line to the curve at the given point.

51. $y = \dfrac{2x}{x + 1}, \quad (1, 1)$

52. $y = \dfrac{\sqrt{x}}{x + 1}, \quad (4, 0.4)$

53. $y = x + \sqrt{x}, \quad (1, 2)$

54. $y = (1 + 2x)^2, \quad (1, 9)$

55. (a) The curve $y = 1/(1 + x^2)$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $(-1, \frac{1}{2})$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

56. (a) The curve $y = x/(1 + x^2)$ is called a **serpentine**. Find an equation of the tangent line to this curve at the point $(3, 0.3)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

57. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find the following values.
(a) $(fg)'(5)$ (b) $(f/g)'(5)$ (c) $(g/f)'(5)$

58. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$, and $g'(3) = 5$, find the following numbers.
(a) $(f + g)'(3)$ (b) $(fg)'(3)$
(c) $\left(\dfrac{f}{g}\right)'(3)$ (d) $\left(\dfrac{f}{f - g}\right)'(3)$

59. If $f(x) = \sqrt{x}\,g(x)$, where $g(4) = 8$ and $g'(4) = 7$, find $f'(4)$.

60. If $h(2) = 4$ and $h'(2) = -3$, find

$$\left.\frac{d}{dx}\left(\frac{h(x)}{x}\right)\right|_{x=2}$$