7.3 Exercises

1. (a) How is the logarithmic function $y = \log_a x$ defined?
(b) What is the domain of this function?
(c) What is the range of this function?
(d) Sketch the general shape of the graph of the function $y = \log_a x$ if $a > 1$.

2. (a) What is the natural logarithm?
(b) What is the common logarithm?
(c) Sketch the graphs of the natural logarithm function and the natural exponential function with a common set of axes.

3–8 Find the exact value of each expression.

3. (a) $\log_{10} 1000$ (b) $\log_{16} 4$

4. (a) $\ln e^{-100}$ (b) $\log_3 81$

5. (a) $\log_5 \frac{1}{25}$ (b) $e^{\ln 15}$

6. (a) $\log_{10} 0.1$ (b) $\log_8 320 - \log_8 5$

7. (a) $\log_{12} 3 + \log_{12} 48$ (b) $\log_2 5 - \log_2 90 + 2\log_2 3$

8. (a) $2^{(\log_2 3 + \log_2 5)}$ (b) $e^{3 \ln 2}$

9–12 Use the properties of logarithms to expand the quantity.

9. $\log_2\left(\dfrac{x^3 y}{z^2}\right)$

10. $\ln \sqrt{a(b^2 + c^2)}$

11. $\ln(uv)^{10}$

12. $\ln \dfrac{3x^2}{(x+1)^5}$

13–18 Express the quantity as a single logarithm.

13. $\log_{10} a - \log_{10} b + \log_{10} c$

14. $\ln(x + y) + \ln(x - y) - 2 \ln z$

15. $2 \ln 4 - \ln 2$

16. $\ln 3 + \frac{1}{3} \ln 8$

17. $\frac{1}{2} \ln x - 5 \ln(x^2 + 1)$

18. $\ln x + a \ln y - b \ln z$

19. Use Formula 7 to evaluate each logarithm correct to six decimal places.
(a) $\log_{12} e$ (b) $\log_6 13.54$ (c) $\log_2 \pi$

20–22 Use Formula 7 to graph the given functions on a common screen. How are these graphs related?

20. $y = \log_2 x$, $y = \log_4 x$, $y = \log_6 x$, $y = \log_8 x$

21. $y = \log_{1.5} x$, $y = \ln x$, $y = \log_{10} x$, $y = \log_{50} x$

22. $y = \ln x$, $y = \log_{10} x$, $y = e^x$, $y = 10^x$

23–28 Make a rough sketch of the graph of each function. Do not use a calculator. Just use the graphs given in Figures 1, 2, and 3 and, if necessary, the transformations of Section 1.3.

23. $y = \log_{10}(x + 5)$

24. $y = \log_2(x - 3)$

25. $y = -\ln x$

26. $y = \ln(10x)$

27. $y = 5 + \ln(x - 2)$

28. $y = \ln|x|$

29–38 Solve each equation for x.

29. (a) $2 \ln x = 1$ (b) $e^{-x} = 5$

30. (a) $e^{2x+3} - 7 = 0$ (b) $\ln(5 - 2x) = -3$

31. (a) $5^{x-3} = 10$ (b) $\log_{10}(x + 1) = 4$

32. (a) $e^{3x+1} = k$ (b) $\log_2(mx) = c$

33. $\ln(\ln x) = 1$

34. $e^{e^x} = 10$

35. $2 \ln x = \ln 2 + \ln(3x - 4)$

36. $\ln(2x + 1) = 2 - \ln x$

37. $e^{ax} = Ce^{bx}$, where $a \neq b$

38. $7e^x - e^{2x} = 12$

39–42 Find the solution of the equation correct to four decimal places.

39. $e^{2+5x} = 100$

40. $\ln(1 + \sqrt{x}) = 2$

41. $\ln(e^x - 2) = 3$

42. $3^{1/(x-4)} = 7$

43–44 Solve each inequality for x.

43. (a) $e^x < 10$ (b) $\ln x > -1$

44. (a) $2 < \ln x < 9$ (b) $e^{2-3x} > 4$

45. Suppose that the graph of $y = \log_2 x$ is drawn on a coordinate grid where the unit of measurement is an inch. How many miles to the right of the origin do we have to move before the height of the curve reaches 3 ft?

46. The velocity of a particle that moves in a straight line under the influence of viscous forces is $v(t) = ce^{-kt}$, where c and k are positive constants.
(a) Show that the acceleration is proportional to the velocity.
(b) Explain the significance of the number c.
(c) At what time is the velocity equal to half the initial velocity?

47. The geologist C. F. Richter defined the magnitude of an earthquake to be $\log_{10}(I/S)$, where I is the intensity of the quake (measured by the amplitude of a seismograph 100 km from the epicenter) and S is the intensity of a "standard" earthquake (where the amplitude is only 1 micron $= 10^{-4}$ cm). The 1989 Loma Prieta earthquake that shook San Francisco had a magnitude of 7.1 on the Richter scale. The 1906 San Francisco earthquake was 16 times as intense. What was its magnitude on the Richter scale?

48. A sound so faint that it can just be heard has intensity $I_0 = 10^{-12}$ watt/m^2 at a frequency of 1000 hertz (Hz). The loudness, in decibels (dB), of a sound with intensity I is then