7.2 Exercises

1. (a) Write an equation that defines the exponential function with base $a > 0$.
   (b) What is the domain of this function?
   (c) If $a \neq 1$, what is the range of this function?
   (d) Sketch the general shape of the graph of the exponential function for each of the following cases.
      (i) $a > 1$ (ii) $a = 1$ (iii) $0 < a < 1$

2. (a) How is the number e defined?
   (b) What is an approximate value for e?
   (c) What is the natural exponential function?

3–6 ▮▮ Graph the given functions on a common screen. How are these graphs related?

3. $y = 2^x$, $y = e^x$, $y = 5^x$, $y = 20^x$

4. $y = e^x$, $y = e^{-x}$, $y = 8^x$, $y = 8^{-x}$

5. $y = 3^x$, $y = 10^x$, $y = \left(\frac{1}{3}\right)^x$, $y = \left(\frac{1}{10}\right)^x$

6. $y = 0.9^x$, $y = 0.6^x$, $y = 0.3^x$, $y = 0.1^x$

7–12 ▮▮ Make a rough sketch of the graph of the function. Do not use a calculator. Just use the graphs given in Figures 3 and 12 and, if necessary, the transformations of Section 1.3.

7. $y = 4^x - 3$ 8. $y = 4^{x-3}$

9. $y = -2^{-x}$ 10. $y = 1 + 2e^x$

11. $y = 3 - e^x$ 12. $y = 2 + 5(1 - e^{-x})$

13. Starting with the graph of $y = e^x$, write the equation of the graph that results from
   (a) shifting 2 units downward
   (b) shifting 2 units to the right
   (c) reflecting about the x-axis
   (d) reflecting about the y-axis
   (e) reflecting about the x-axis and then about the y-axis

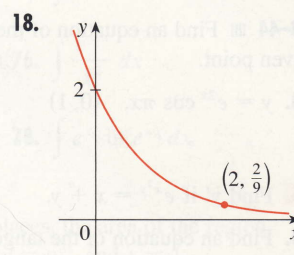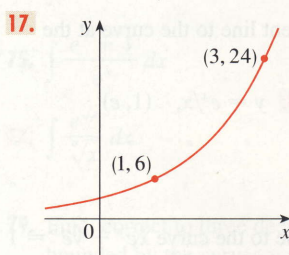14. Starting with the graph of $y = e^x$, find the equation of the graph that results from
   (a) reflecting about the line $y = 4$
   (b) reflecting about the line $x = 2$

15–16 ▮▮ Find the domain of each function.

15. (a) $f(x) = \dfrac{1}{1 + e^x}$ (b) $f(x) = \dfrac{1}{1 - e^x}$

16. (a) $g(t) = \sin(e^{-t})$ (b) $g(t) = \sqrt{1 - 2^t}$

17–18 ▮▮ Find the exponential function $f(x) = Ca^x$ whose graph is given.

17.

18.

19. Suppose the graphs of $f(x) = x^2$ and $g(x) = 2^x$ are drawn on a coordinate grid where the unit of measurement is 1 inch. Show that, at a distance 2 ft to the right of the origin, the height of the graph of f is 48 ft but the height of the graph of g is about 265 mi.

20. Compare the rates of growth of the functions $f(x) = x^5$ and $g(x) = 5^x$ by graphing both functions in several viewing rectangles. Find all points of intersection of the graphs correct to one decimal place.

21. Compare the functions $f(x) = x^{10}$ and $g(x) = e^x$ by graphing both f and g in several viewing rectangles. When does the graph of g finally surpass the graph of f?

22. Use a graph to estimate the values of x such that $e^x > 1,000,000,000$.

23–28 ▮▮ Find the limit.

23. $\lim\limits_{x \to \infty} (1.001)^x$ 24. $\lim\limits_{x \to \infty} e^{-x^2}$

25. $\lim\limits_{x \to \infty} \dfrac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$ 26. $\lim\limits_{x \to (\pi/2)^+} e^{\tan x}$

27. $\lim\limits_{x \to 2^+} e^{3/(2-x)}$ 28. $\lim\limits_{x \to 2^-} e^{3/(2-x)}$

29–42 ▮▮ Differentiate the function.

29. $f(x) = x^2 e^x$ 30. $y = \dfrac{e^x}{1 + x}$

31. $y = e^{ax^3}$ 32. $y = e^u (\cos u + cu)$

33. $f(u) = e^{1/u}$ 34. $g(x) = \sqrt{x}\, e^x$

35. $F(t) = e^{t \sin 2t}$ 36. $y = e^{k \tan \sqrt{x}}$

37. $y = \sqrt{1 + 2e^{3x}}$ 38. $y = \cos(e^{\pi x})$