7.1 Exercises

1. (a) What is a one-to-one function?
   (b) How can you tell from the graph of a function whether it is one-to-one?

2. (a) Suppose f is a one-to-one function with domain A and range B. How is the inverse function f^{-1} defined? What is the domain of f^{-1}? What is the range of f^{-1}?
   (b) If you are given a formula for f, how do you find a formula for f^{-1}?
   (c) If you are given the graph of f, how do you find the graph of f^{-1}?

3–16 ||| A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

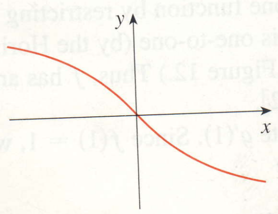
3.

x	1	2	3	4	5	6
$f(x)$	1.5	2.0	3.6	5.3	2.8	2.0

4.

x	1	2	3	4	5	6
$f(x)$	1	2	4	8	16	32

5.



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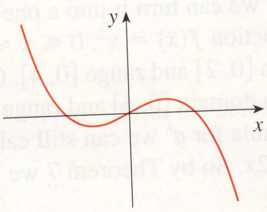

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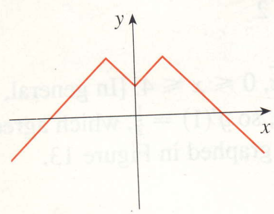

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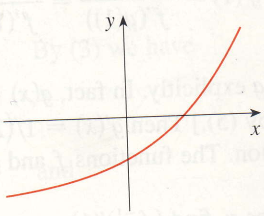


9. $f(x) = \frac{1}{2}(x + 5)$
10. $f(x) = 1 + 4x - x^2$
11. $g(x) = \sqrt{x}$
12. $g(x) = |x|$
13. $h(x) = x^4 + 5$
14. $h(x) = x^4 + 5, \quad 0 \leqslant x \leqslant 2$
15. $f(t)$ is the height of a football t seconds after kickoff.
16. $f(t)$ is your height at age t.

17–18 ||| Use a graph to decide whether f is one-to-one.

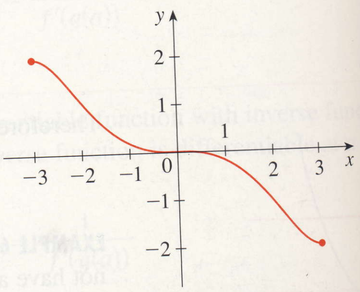
17. $f(x) = x^3 - x$
18. $f(x) = x^3 + x$

19. If f is a one-to-one function such that $f(2) = 9$, what is $f^{-1}(9)$?

20. If $f(x) = x + \cos x$, find $f^{-1}(1)$.

21. If $h(x) = x + \sqrt{x}$, find $h^{-1}(6)$.

22. The graph of f is given.
    (a) Why is f one-to-one?
    (b) State the domain and range of f^{-1}.
    (c) Estimate the value of $f^{-1}(1)$.



23. The formula $C = \frac{5}{9}(F - 32)$, where $F \geqslant -459.67$, expresses the Celsius temperature C as a function of the Fahrenheit temperature F. Find a formula for the inverse function and interpret it. What is the domain of the inverse function?

24. In the theory of relativity, the mass of a particle with speed v is
$$m = f(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$
where m_0 is the rest mass of the particle and c is the speed of light in a vacuum. Find the inverse function of f and explain its meaning.

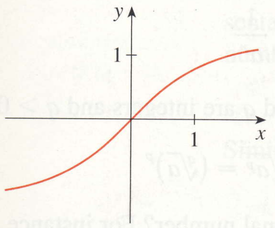
25–30 ||| Find a formula for the inverse of the function.

25. $f(x) = 3 - 2x$
26. $f(x) = \dfrac{4x - 1}{2x + 3}$
27. $f(x) = \sqrt{10 - 3x}$
28. $y = 2x^3 + 3$
29. $y = \dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$
30. $f(x) = 2x^2 - 8x, \quad x \geqslant$

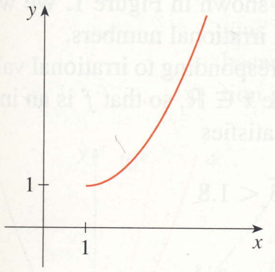
31–32 ||| Find an explicit formula for f^{-1} and use it to graph f^{-1}, f, and the line $y = x$ on the same screen. To check your work, see whether the graphs of f and f^{-1} are reflections about the line.

31. $f(x) = 1 - 2/x^2, \quad x > 0$
32. $f(x) = \sqrt{x^2 + 2x}, \quad x >$

33. Use the given graph of f to sketch the graph of f^{-1}.



34. Use the given graph of f to sketch the graphs of f^{-1} and $1/f$.



35–38 |||

(a) Show that f is one-to-one.
(b) Use Theorem 7 to find $g'(a)$, where $g = f^{-1}$.
(c) Calculate $g(x)$ and state the domain and range of g.
(d) Calculate $g'(a)$ from the formula in part (c) and check that it agrees with the result of part (b).
(e) Sketch the graphs of f and g on the same axes.

35. $f(x) = x^3, \quad a = 8$

36. $f(x) = \sqrt{x - 2}, \quad a = 2$

37. $f(x) = 9 - x^2, \quad 0 \leqslant x \leqslant 3, \quad a = 8$

38. $f(x) = 1/(x - 1), \quad x > 1, \quad a = 2$

39–42 ||| Find $(f^{-1})'(a)$.

39. $f(x) = x^3 + x + 1, \quad a = 1$

40. $f(x) = x^5 - x^3 + 2x, \quad a = 2$

41. $f(x) = 3 + x^2 + \tan(\pi x/2), \quad -1 < x < 1, \quad a = 3$

42. $f(x) = \sqrt{x^3 + x^2 + x + 1}, \quad a = 2$

43. Suppose g is the inverse function of f and $f(4) = 5$, $f'(4) = \frac{2}{3}$. Find $g'(5)$.

44. Suppose g is the inverse function of a differentiable function f and let $G(x) = 1/g(x)$. If $f(3) = 2$ and $f'(3) = \frac{1}{9}$, find $G'(2)$.

CAS **45.** Use a computer algebra system to find an explicit expression for the inverse of the function $f(x) = \sqrt{x^3 + x^2 + x + 1}$. (Your CAS will produce three possible expressions. Explain why two of them are irrelevant in this context.)

46. Show that $h(x) = \sin x$, $x \in \mathbb{R}$, is not one-to-one, but its restriction $f(x) = \sin x$, $-\pi/2 \leqslant x \leqslant \pi/2$, is one-to-one. Compute the derivative of $f^{-1} = \sin^{-1}$ by the method of Note 2.

47. (a) If we shift a curve to the left, what happens to its reflection about the line $y = x$? In view of this geometric principle, find an expression for the inverse of $g(x) = f(x + c)$, where f is a one-to-one function.
(b) Find an expression for the inverse of $h(x) = f(cx)$, where $c \neq 0$.

48. (a) If f is a one-to-one, twice differentiable function with inverse function g, show that

$$g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}$$

(b) Deduce that if f is increasing and concave upward, then its inverse function is concave downward.

7.2 Exponential Functions and Their Derivatives

If your instructor has assigned Sections 7.2*, 7.3*, and 7.4*, you don't need to read Sections 7.2–7.4 (pp. 421–450).

The function $f(x) = 2^x$ is called an *exponential function* because the variable, x, is the exponent. It should not be confused with the power function $g(x) = x^2$, in which the variable is the base.

In general, an **exponential function** is a function of the form

$$f(x) = a^x$$

where a is a positive constant. Let's recall what this means.

If $x = n$, a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$$