

- (b) Draw a diagram illustrating the general situation. Introduce notation and label the diagram with your symbols.
- (c) Write an expression for the volume.
- (d) Use the given information to write an equation that relates the variables.
- (e) Use part (d) to write the volume as a function of one variable.
- (f) Finish solving the problem and compare the answer with your estimate in part (a).
9. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?
10. A box with a square base and open top must have a volume of  $32,000 \text{ cm}^3$ . Find the dimensions of the box that minimize the amount of material used.
11. If  $1200 \text{ cm}^2$  of material is available to make a box with a square base and an open top, find the largest possible volume of the box.
12. A rectangular storage container with an open top is to have a volume of  $10 \text{ m}^3$ . The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.
13. Do Exercise 12 assuming the container has a lid that is made from the same material as the sides.
14. (a) Show that of all the rectangles with a given area, the one with smallest perimeter is a square.  
(b) Show that of all the rectangles with a given perimeter, the one with greatest area is a square.
15. Find the point on the line  $y = 4x + 7$  that is closest to the origin.
16. Find the point on the line  $6x + y = 9$  that is closest to the point  $(-3, 1)$ .
17. Find the points on the ellipse  $4x^2 + y^2 = 4$  that are farthest away from the point  $(1, 0)$ .
18. Find, correct to two decimal places, the coordinates of the point on the curve  $y = \tan x$  that is closest to the point  $(1, 1)$ .
19. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius  $r$ .
20. Find the area of the largest rectangle that can be inscribed in the ellipse  $x^2/a^2 + y^2/b^2 = 1$ .
21. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side  $L$  if one side of the rectangle lies on the base of the triangle.
22. Find the dimensions of the rectangle of largest area that has its base on the  $x$ -axis and its other two vertices above the  $x$ -axis and lying on the parabola  $y = 8 - x^2$ .
23. Find the dimensions of the isosceles triangle of largest area that can be inscribed in a circle of radius  $r$ .
24. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs.
25. A right circular cylinder is inscribed in a sphere of radius  $r$ . Find the largest possible volume of such a cylinder.
26. A right circular cylinder is inscribed in a cone with height  $h$  and base radius  $r$ . Find the largest possible volume of such a cylinder.
27. A right circular cylinder is inscribed in a sphere of radius  $r$ . Find the largest possible surface area of such a cylinder.
28. A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus, the diameter of the semicircle is equal to the width of the rectangle. See Exercise 52 on page 24.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.
29. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at  $384 \text{ cm}^2$ , find the dimensions of the poster with the smallest area.
30. A poster is to have an area of  $180 \text{ in}^2$  with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area?
31. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) A minimum?
32. Answer Exercise 31 if one piece is bent into a square and the other into a circle.
33. A cylindrical can without a top is made to contain  $V \text{ cm}^3$  of liquid. Find the dimensions that will minimize the cost of the metal to make the can.
34. A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?
35. A cone-shaped drinking cup is made from a circular piece of paper of radius  $R$  by cutting out a sector and joining the edges  $CA$  and  $CB$ . Find the maximum capacity of such a cup.

