Assignment # 9.

Due Mar. 24, 17:00

Problem 1. Is the following (improper) integral convergent?

a. $\displaystyle\int_1^\infty \frac{\arctan x}{x^2}\,dx$ **b.** $\displaystyle\int_0^1 \frac{\sin x}{x^2}\,dx$

Problem 2. Find the following integral (or explain why it does not exist)

a. $\displaystyle\int_0^1 \ln x\,dx$ **b.** $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$ **c.** $\displaystyle\int_{-1}^1 \frac{1}{x^2}\,dx$

Problem 3. Let $a \in \mathbb{R}$. Let f, g be non-negative functions defined on $[a, \infty)$. Assume that for every $b \in [a, \infty)$ the functions f, g are bounded and integrable on $[a, b]$. Assume also that

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L < \infty \qquad \text{and} \qquad \int_a^\infty g(x)\,dx \text{ is convergent.}$$

Prove that

$$\int_a^\infty f(x)\,dx \text{ is convergent.}$$