Assignment # 5.

Due Feb. 24, 17:00

Problem 1. Differentiate

a. $\displaystyle\int_{-10}^{x} e^t\,dt,$ **b.** $\displaystyle\int_{2}^{x^2} \ln t\,dt,$ **c.** $\displaystyle\int_{\sin x}^{4} t^2\,dt,$ **d.** $\displaystyle\int_{\tan x}^{\frac{1}{x}} \frac{1}{t}\,dt,$

Problem 2. Let f be a bounded integrable non-negative function on $[a,b]$. Is it true that

a. $\int_a^b f(x)\,dx = 0$ implies $f(x) = 0$ for every x.

b. $\int_a^b f(x)\,dx = 0$ and f is continuous on $[a,b]$ implies $f(x) = 0$ for every x.

Problem 3. Assume that $\lim\limits_{x\to a} f(x) = L > 0$ and $\lim\limits_{x\to a} g(x) = M$. Prove that $\lim\limits_{x\to a} f(x)^{g(x)} = L^M$.

Problem 4. Find domains of the following functions.

a. $f(x) = \log_2(x-3)+\log_7(5-x),$ **b.** $g(x) = \log_2 \log_3 \log_4 x,$

c. $f(x) = \left(\log_{\sqrt{3}} \tan x\right)^{\pi},$

Problem 5. Let $a > 0$, $x \in \mathbb{R}$. Prove that

$$a^{-x} = \frac{1}{a^x}$$

(you may use that this holds for $x \in \mathbb{Q}$ and facts proved in class).