Assignment # 1.

Due Jan. 20, 17:00

Problem 1. Does $f'(a)$ exists? Explain your answer.

a. $a = 0, \quad f(x) = \begin{cases} 0, & \text{if } x \geq 0, \\ x^2, & \text{if } x < 0; \end{cases}$

b. $a = 0, \quad f(x) = \begin{cases} -x, & \text{if } x \geq 0, \\ x^2, & \text{if } x < 0; \end{cases}$

c. $a = 0, \quad f(x) = \begin{cases} -x^2, & \text{if } x \geq 0, \\ x^2, & \text{if } x < 0; \end{cases}$

d. $a = 0, \quad f(x) = \begin{cases} \sin x, & \text{if } x \geq 0, \\ x, & \text{if } x < 0; \end{cases}$

e. $a = 2, \quad f(x) = \begin{cases} x+1, & \text{if } x \geq 0, \\ x-1, & \text{if } x < 0; \end{cases}$

f. $a = 1, \quad f(x) = \begin{cases} 1-x, & \text{if } x \geq 0, \\ (1-x)(2-x), & \text{if } x < 0; \end{cases}$

Problem 2. Differentiate (don't simplify)

a. $\sin(x^2+2x+1)$, **b.** $\cot(1/x)$, **c.** $\dfrac{x^2}{x+1}$, **d.** $\sqrt{x+\sqrt{x}}$.

Problem 3. Using Lagrange Mean Value Theorem prove that for every $a, b \in \mathbb{R}$ one has $|\sin a - \sin b| \leq |a - b|$.

Problem 4. A number a is called a fixed point of a function f if $f(a) = a$. Assuming that f is differentiable and that for every x one has $f'(x) \neq 1$ prove that f has at most one fixed point.

Problem 5. Using differentiation prove that for every $x \in \mathbb{R}$ one has

$$\cos x \geq 1 - \frac{x^2}{2}.$$