FINAL EXAM

MATH 117, FALL 2007

Closed books, closed notes. You are allowed to refer to results that have appeared in class or in homework as long as you cite the result (except when the entire problem was done in class or in homework). Please write your solutions carefully. Provide as many details as possible. Use one examination booklet as scratch paper, and submit the other one. Good luck.

Problem 1 (20 pt)**.** Write the following definitions:

(a) a convergent sequence
(b) the Completeness Axiom

Problem 2 (10 pt)**.** Prove that if f is differentiable at a then f is continuous at a.

Problem 3 (10 pt)**.** State and prove the Maximal Principle.

Problem 4 (20 pt)**.** Compute (and explain):

$$\lim \frac{100n^5 \sin n}{3^n + 3n^2 + 1} \qquad \lim_{x\to 2} \frac{2x^2 - 3x + \cos \pi x}{x^3 + 1} \qquad \lim_{x\to 0^-} \frac{|x|}{x} \qquad \Big(\cos^5\big(\frac{x^2}{x-1}\big)\Big)'$$

Problem 5 (10 pt)**.** Given $f(x) = \begin{cases} x \cos \frac{1}{x} & \text{if } x \neq 0; \\ 0 & \text{if } x = 0. \end{cases}$

(a) Determine whether f is continuous at 0.
(b) Is it differentiable at 0?

Problem 6 (10 pt)**.** *True or false:* $\sup(x_n + y_n) = \sup x_n + \sup y_n$ for any two sequences (x_n) and (y_n).

Problem 7 (10 pt)**.** Let $x_n = \sin n$. Prove that (x_n) has an accumulation point.

Problem 8 (10 pt)**.** Prove that if $\lim_{x\to a^-} f(x) = +\infty$ then $\lim_{x\to a^-} \frac{1}{f(x)} = 0$.

Problem 9 (bonus: 10 pt)**.** Suppose that A is an infinite set of positive real numbers such that the sum of every finite subset of A is less than 1. Show that A is countable. (*Hint:* for every $n \in \mathbb{N}$, put $A_n = \{x \in A : x > \frac{1}{n}\}$.)