Quiz # 6

Problem 2. Is the following sequence convergent? If YES find the limit, if NOT explain why.

$$\left\{\left(1+\frac{1}{n}\right)^{2n+1}\right\}_{n=1}^{\infty}.$$

Solution. Note that in class we proved that

$$\left\{\left(1+\frac{1}{n}\right)^{n}\right\}_{n=1}^{\infty}$$

is convergent and we denoted its limit by e. Clearly,

$$\left(1+\frac{1}{n}\right)^{2n+1} = \left(1+\frac{1}{n}\right)^{n}\left(1+\frac{1}{n}\right)^{n}\left(1+\frac{1}{n}\right).$$

Thus, applying theorem about the limit of the product of sequences we obtain that the limit of the given sequence exists and

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n+1} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\lim_{n\to\infty}\left(1+\frac{1}{n}\right) = e^2$$

(we used also that $1/n \to 0$, so $(1+1/n) \to 1$ as $n \to \infty$). $\square$

Answer.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n+1} = e^2.$$