Quiz # 5

Problem 2. Find the limit of the following sequence and prove your answer using only the definition.

$$\left\{ \frac{n^2 + n + 4}{n^2 + 2} \right\}_{n=1}^{\infty}.$$

Solution. We show that the limit is 1. Fix an arbitrary $\varepsilon > 0$. Choose $N > 3/\varepsilon$. Then for every $n \geq N$ we have

$$\left| \frac{n^2 + n + 4}{n^2 + 2} - 1 \right| = \frac{n + 2}{n^2 + 2} \leq \frac{n + 2}{n^2} \leq \frac{3n}{n^2} = \frac{3}{n} \leq \frac{3}{N} < \varepsilon.$$

It proves the result. $\square$

Answer.

$$\lim_{n \to \infty} \frac{n^2 + n + 4}{n^2 + 2} = 1.$$