Quiz # 4

Problem 1. Give the definitions of an upper bound, the supremum, and the maximum of a bounded above non-empty set.

Solution. Let S be a bounded above non-empty set (it means that there exists an upper bound). A number $a \in \mathbb{R}$ is called an upper bound of S if a is larger than or equal to any number in S (in other words $\forall x \in S \quad x \leq a$). The smallest upper bound is called supremum. If supremum belongs to the set S then it is called maximum. $\square$

Problem 2. Solve and write the answer in the interval notation

$$\left| \frac{2}{1-x} \right| > 4.$$

Solution. First note that for $x = 1$ the fraction $\frac{2}{1-x}$ is not defined (and thus $x = 1$ is not a solution). Now we consider the case $x \neq 1$ (and all equivalences will be under this condition).

$$\left| \frac{2}{1-x} \right| = \frac{2}{|x-1|} > 4 \;\Leftrightarrow\; \frac{1}{2} > |x-1|\,.$$

Way 1. We consider 2 cases.
Case 1. $x - 1 > 0$ (in particular, $x \neq 1$). In this case $|x-1| = x-1$, so we have

$$\frac{1}{2} > x - 1 \;\Leftrightarrow\; \frac{3}{2} > x.$$

Since we consider the case $x > 1$ the answer for this case is $x \in (1, 3/2)$.
Case 2. $x - 1 < 0$ (in particular, $x \neq 1$). In this case $|x-1| = -(x-1)$, so we have

$$\frac{1}{2} > -x + 1 \;\Leftrightarrow\; x > \frac{1}{2}.$$

Since we consider the case $x < 1$ the answer for this case is $x \in (1/2, 1)$.

Combining two cases we obtain $x \in (1/2, 1) \cup (1, 3/2)$. (Note again that the point $x = 1$ is excluded).
Way 2. We apply fact saying that $|y| < r$ if and only if $-r < y < r$.

$$\frac{1}{2} > |x-1| \;\Leftrightarrow\; -\frac{1}{2} < x - 1 < \frac{1}{2} \;\Leftrightarrow\; \frac{1}{2} < x < \frac{3}{2}.$$

Combining this with $x \neq 1$ we obtain $x \in (1/2, 1) \cup (1, 3/2)$. $\square$

Answer. $(1/2, 1) \cup (1, 3/2)$.
Remarks. **1.** Answer can be also written as $(1/2, 3/2) \setminus \{1\}$.
2. Another way to solve the problem is to use that

$$|y| > r \qquad \text{if and only if} \qquad \big(\text{ either } \; y > 4 \;\text{ or }\; y < -4\big).$$