Answers to drill problems 6.

Problem 1.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}}=1, \qquad \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2}=\infty.$$

Problem 2. For example, 2 is a bound.

Problem 3.

a. $\lim\limits_{x\to 0} x^n = 0, \qquad \lim\limits_{x\to\infty} x^n = \infty \qquad \lim\limits_{x\to-\infty} x^{2m} = \infty, \qquad \lim\limits_{x\to\infty} x^{2m+1} = \infty.$

b. $\lim\limits_{x\to 0} \dfrac{1}{x^n}$ does not exists, $\qquad \lim\limits_{x\to\infty} \dfrac{1}{x^n} = 0 \qquad \lim\limits_{x\to-\infty} \dfrac{1}{x^n} = 0.$

c. $\lim\limits_{x\to 0} |x| = 0, \qquad \lim\limits_{x\to\infty} |x| = \infty, \qquad \lim\limits_{x\to-\infty} |x| = \infty.$

d. $\lim\limits_{x\to 0} \dfrac{3x^2+2x-1}{7x-x^2+5} = -\dfrac{1}{5}, \qquad \lim\limits_{x\to\infty} \dfrac{3x^2+2x-1}{7x-x^2+5} = -3, \qquad \lim\limits_{x\to-\infty} \dfrac{3x^2+2x-1}{7x-x^2+5} = -3.$

e. $\lim\limits_{x\to 0} \dfrac{x^5}{2^x} = 0, \qquad \lim\limits_{x\to\infty} \dfrac{x^5}{2^x} = 0, \qquad \lim\limits_{x\to-\infty} \dfrac{x^5}{2^x} = -\infty.$

Problem 4.

a. The limit doesn not exist.

b. $\lim\limits_{x\to 0} x\sin\dfrac{1}{x} = 0$

Problem 5.

$$\lim_{x\to 4}\frac{\sqrt{1+2x}\ -3}{\sqrt{x}\ -2}=\frac{4}{3}.$$