Answers to drill problems 2.

Problem 1. Let A, B be sets. Prove that $A\Delta B = (A \cup B) \setminus (A \cap B)$.

Remark. Note that by the definition $A\Delta B = (A \setminus B) \cup (B \setminus A)$. Thus you have to prove that

$$\Big((A \setminus B) \cup (B \setminus A)\Big) \subset \Big((A \cup B) \setminus (A \cap B)\Big) \quad \text{and} \quad \Big((A \cup B) \setminus (A \cap B)\Big) \subset \Big((A \setminus B) \cup (B \setminus A)\Big).$$

So you have to start with an arbitrary $x \in (A \setminus B) \cup (B \setminus A)$ and show that $x \in (A \cup B) \setminus (A \cap B)$. Then you should take an arbitrary $x \in (A \cup B) \setminus (A \cap B)$ and show that $x \in (A \setminus B) \cup (B \setminus A)$

Problem 2. Is the statement below true or false? Write its negation. Is the negation true or false? Why?

a. $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{N} \quad y = x^2$, **b.** $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{N} \quad y = 5 - x^2$,
c. $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{N} \quad y = 3$, **d.** $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{N} \quad x = 0$,
e. $\forall x \in \mathbb{N}\ \exists y \in \mathbb{Z} \quad y^2 - x = 0$, **f.** $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z} \quad y + x \neq 0$,
g. $\exists x \in \mathbb{N}\ \forall y \in \mathbb{Q} \quad x + y \in \mathbb{N}$, **h.** $\exists x \in \mathbb{N}\ \forall y \in \mathbb{Q} \quad x + y \neq 0$,
i. $\exists x \in \mathbb{N}\ \forall y \in \mathbb{Q} \quad x = 10$, **j.** $\exists x \in \mathbb{N}\ \forall y \in \mathbb{Q} \quad y = 5$,
k. $\exists x \in \mathbb{N}\ \forall y \in \mathbb{Z} \quad y = x^2$, **l.** $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{Z} \quad x + y = 0$.

Answer.
a. False (take $x = 0$) . The negation (which is true) is $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{N} \quad y \neq x^2$,
b. False (take $x = 5$). The negation (which is true) is $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{N} \quad y \neq 5 - x^2$,
c. True (take $y = 3$). The negation (which is false) is $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{N} \quad y \neq 3$,
d. False (take $x = 2$). The negation (which is true) is $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{N} \quad x \neq 0$,
e. False (take $x = 2$). The negation (which is true) is $\exists x \in \mathbb{N}\ \forall y \in \mathbb{Z} \quad y^2 - x \neq 0$,
f. True (take $y = 1 - x$). The negation (which is false) is $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{Z} \quad y + x = 0$,
g. False (take $y = 1/2$). The negation (which is true) is $\forall x \in \mathbb{N}\ \exists y \in \mathbb{Q} \quad x + y \notin \mathbb{N}$,
h. False (take $y = -x$). The negation (which is true) is $\forall x \in \mathbb{N}\ \exists y \in \mathbb{Q} \quad x + y = 0$,
i. True (take $x = 10$). The negation (which is false) is $\forall x \in \mathbb{N}\ \exists y \in \mathbb{Q} \quad x \neq 10$,
j. False (take $y = 2$). The negation (which is true) is $\forall x \in \mathbb{N}\ \exists y \in \mathbb{Q} \quad y \neq 5$,
k. False (take $y = x^2 + 1$). The negation (which is true) is $\forall x \in \mathbb{N}\ \exists y \in \mathbb{Z} \quad y \neq x^2$,
l. False (take $y = 1 - x$). The negation (which is true) is $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z} \quad x + y \neq 0$.

Remark. Note that **c** says $\exists y \in \mathbb{N} \quad y = 3$; **d** says $\forall x \in \mathbb{Z} \quad x = 0$; **i** says $\exists x \in \mathbb{N} \quad x = 10$; **j** says $\forall y \in \mathbb{Q} \quad y = 5$.