Answers to drill problems 1.

Problem 1. Find the sets $A \cap B$, $A \cup B$, $A \setminus B$, $A \Delta B$, where
a. $A = \{1, 3, 7\}$, $B = \{2, 3, 5\}$, **b.** $A = \{4, 6\}$, $B = \{4, 6\}$,
c. $A = \emptyset$, $B = \{2, 4\}$, **d.** $A = \{2, 4\}$, $B = \emptyset$,
e. $A = \mathbb{N}$, $B = \{2, 4, 6, 8, ...\}$, **f.** $A = \{1, 3, 5, 7, ...\}$, $B = \mathbb{N}$,
g. $A = \{\emptyset, \{1\}\}$, $B = \{1\}$.

Answer.
a. $A \cap B = \{3\}$, $A \cup B = \{1, 2, 3, 5, 7\}$, $A \setminus B = \{1, 7\}$, $A \Delta B = \{1, 2, 5, 7\}$;
b. $A \cap B = \{4, 6\}$, $A \cup B = \{4, 6\}$, $A \setminus B = \emptyset$, $A \Delta B = \emptyset$;
c. $A \cap B = \emptyset$, $A \cup B = \{2, 4\}$, $A \setminus B = \emptyset$, $A \Delta B = \{2, 4\}$;
d. $A \cap B = \emptyset$, $A \cup B = \{2, 4\}$, $A \setminus B = \{2, 4\}$, $A \Delta B = \{2, 4\}$;
e. $A \cap B = \{2, 4, 6, 8, ...\}$, $A \cup B = \mathbb{N}$, $A \setminus B = \{1, 3, 5, 7, ...\}$,
$A \Delta B = \{1, 3, 5, 7, ...\}$;
f. $A \cap B = \{1, 3, 5, 7, ...\}$, $A \cup B = \mathbb{N}$, $A \setminus B = \emptyset$, $A \Delta B = \{2, 4, 6, 8, ...\}$;
g. $A \cap B = \emptyset$, $A \cup B = \{1, \emptyset, \{1\}\}$, $A \setminus B = \{\emptyset, \{1\}\}$, $A \Delta B = \{1, \emptyset, \{1\}\}$.

Problem 2. Find the power set $\mathcal{P}(A)$, where
a. $A = \{x, y, z\}$, **b.** $A = \emptyset$.

Answer.
a. $\mathcal{P}(\{x, y, z\}) = \{\emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$,
b. $\mathcal{P}(\emptyset) = \{\emptyset\}$.

Problem 3. Let P, Q, R be statements. Write truth tables for
a. $P_1 = (P$ and $(Q$ or $R))$, **b.** $P_2 = ((P$ and $Q)$ or $R)$,
c. $P_3 = (P$ or $(Q$ and $R))$, **d.** $P_4 = ((P$ or $Q)$ and $R)$,
e. $P_5 = ((P$ and $Q)$ or $(P$ and $R))$, **f.** $P_6 = ((P$ or $Q)$ and $(Q$ or $R))$,
g. $P_7 = ($not $(P$ or $Q))$, **h.** $P_8 = ((\text{not } P)$ and $(\text{not } Q))$,
i. $P_9 = ((P \Rightarrow (\text{not } Q)) \Rightarrow R)$, **j.** $P_{10} = (P \Rightarrow ((\text{not } Q) \Rightarrow R))$.
What statements are equal to each other?

Answer. (See the attached table). From the table (or one can check from definitions and logic operations) we see that $P_1 = P_5, P_2 = P_9, P_7 = P_8$. Note also that $P_2 = ((P$ or $R)$ and $(P$ or $Q))$, $P_6 = Q$ or $(P$ and $R)$.