

Drill Problems # 6.

Problem 1. Find the following limits (if exists):

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}}, \quad \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2}.$$

Problem 2. Prove that $f(x) = \frac{1+x^2}{1+x^4}$ is a bounded function on \mathbb{R} .

Problem 3. Find limits of the following functions at 0 , ∞ , $-\infty$ (here n is a fixed natural number):

a. $f(x) = x^n$, **b.** $f(x) = \frac{1}{x^n}$, **c.** $f(x) = |x|$,

d. $f(x) = \frac{3x^2 + 2x - 1}{7x - x^2 + 5}$, **e.** $f(x) = \frac{x^5}{2^x}$.

Problem 4. Find limits of the following functions at 0 (if exist):

a. $f(x) = \sin \frac{1}{x}$, **b.** $f(x) = x \sin \frac{1}{x}$.

Problem 5. Find the following limit (if exists):

$$\lim_{x \rightarrow 4} \frac{\sqrt{1+2x} - 3}{\sqrt{x} - 2}.$$